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A T-odd asymmetry in neutralino production and decay

Alfred Bartl, Stefan Hesselbach and Karl Hohenwarter-Sodek
Institut für Theoretische Physik, Universität Wien
A-1090 Vienna, Austria
E-mail: bartl@ap.univie.ac.at, stefan.hesselbach@univie.ac.at, khsodek@ap.univie.ac.at

Hans Fraas
Institut für Theoretische Physik und Astrophysik, Universität Würzburg
Am Hubland, D-97074 Würzburg, Germany
E-mail: fraas@physik.uni-wuerzburg.de

Gudrid Moortgat-Pick
IPPP, University of Durham
Durham DH1 3LE, U.K.
E-mail: g.a.moortgat-pick@durham.ac.uk

Abstract: We study CP-violating effects in neutralino production and subsequent decay within the Minimal Supersymmetric Standard Model with complex parameters M_1 and μ. The observable we propose is a T-odd asymmetry based on a triple product in neutralino production $e^+e^- \to \tilde{\chi}^0_i \tilde{\chi}^0_2$, $i = 1, \ldots, 4$, with subsequent leptonic three-body decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \ell^+ \ell^-$, $\ell = e, \mu$, at an e^+e^- linear collider with $\sqrt{s} = 500\,\mathrm{GeV}$ and polarised beams. We provide compact analytical formulae for the cross section and the T-odd asymmetry taking into account the complete spin correlations between production and decay. We give numerical predictions for the cross section and the T-odd asymmetry. The asymmetry can go up to 10 %.

Keywords: Supersymmetry Phenomenology, Supersymmetric Standard Model, CP violation.

Contents

1. Introduction

In the Standard Model (SM) the phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix is the only source of CP violation. The small amount of CP violation in the SM, however, is not sufficient to explain the baryon-antibaryon asymmetry of the universe [1]. The lagrangian of the Minimal Supersymmetric Standard Model (MSSM) contains several complex parameters, which can give rise to new CP-violating phenomena [2]. In the neutralino sector of the MSSM two complex parameters appear, which lead to CP-violating effects in neutralino production and decay. These parameters are the U(1) gaugino mass parameter M_1 and the higgsino mass parameter μ. The phase of the SU(2) gaugino mass parameter M_2 can be eliminated by an U(1)-symmetry of the model.

One of the main goals of a future e^+e^- linear collider will be a careful study of the properties of supersymmetric (SUSY) particles [3]. The neutralinos $\tilde{\chi}_i^0$, $i = 1, \ldots, 4$, will be particularly interesting, because the lightest neutralino $\tilde{\chi}_1^0$ is expected to be the lightest

SUSY particle (LSP), which is stable if R-parity is conserved. The second lightest neutralino, $\tilde{\chi}^0_2$, will presumably be among the lightest visible SUSY particles. Therefore, the study of production and decay of the neutralinos $\tilde{\chi}^0_i$ [3]–[5] and a precise determination of the underlying supersymmetric parameters M_1, M_2, μ and $\tan\beta$ including the phases ϕ_{M_1} and ϕ_μ of M_1 and μ will play an important role at future linear colliders. The phases of M_1 and μ cause CP-violating effects already at tree level. Therefore these effects could be large and thus be measurable at a high luminosity e^+e^- linear collider. Methods to determine these parameters in neutralino and chargino production have been presented in [6]–[9]. In particular the method of [9] is based on the analysis of the masses and production cross sections of only the light neutralinos $\tilde{\chi}^0_1$ and $\tilde{\chi}^0_2$ and the light chargino $\tilde{\chi}^\pm_1$ and will allow the determination of the parameters $|M_1|$, M_2, $|\mu|$ and the phases ϕ_{M_1} and ϕ_μ. All these methods involve only CP-even quantities, in which the signs of the phases ϕ_{M_1} and ϕ_μ can not be determined. For an unambiguous determination of ϕ_{M_1} and ϕ_μ including their signs one has to rely on CP-sensitive observables.

The phases of the complex parameters are constrained or correlated by the experimental upper limits on the electric dipole moments of electron, neutron and the atoms ^{199}Hg and ^{205}Tl [10]. In a constrained MSSM the restrictions on the phases can be rather severe. However, there may be cancellations between the contributions of different complex parameters, which allow larger values for the phases [11]. For example, in a constrained MSSM and if substantial cancellations are present, ϕ_μ is restricted to $|\phi_\mu| \lesssim 0.1\pi$, whereas ϕ_{M_1} and ϕ_A, the phase of the trilinear scalar coupling parameter, turn out to be essentially unconstrained, but correlated with ϕ_μ [12]. Moreover, the restrictions are very model dependent. For example, when also lepton flavour violating terms are included, then the restriction on ϕ_μ may disappear [13]. Therefore it is necessary to determine the phases in an unambiguous way by measurements of CP-odd observables.

A useful tool to study these CP-violating effects are T-odd observables, based on triple products of momenta or spin vectors of the particles involved [14, 15]. In this paper we study a T-odd asymmetry in neutralino production

$$e^+e^- \to \tilde{\chi}^0_k \tilde{\chi}^0_2 \,, \qquad k = 1, \dots, 4 \,, \tag{1.1}$$

with subsequent leptonic three-body decay

$$\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \ell^+ \ell^- \,, \tag{1.2}$$

with $\ell = e, \mu$. Using the triple product of the initial electron momentum $\vec{p}_{e^-}$ and the two final lepton momenta $\vec{p}_{\ell^+}$ and $\vec{p}_{\ell^-}$,

$$O_T = \vec{p}_{\ell^+} \cdot (\vec{p}_{\ell^-} \times \vec{p}_{e^-}) \,, \tag{1.3}$$

we define a T-odd asymmetry

$$A_T = \frac{\int \mathrm{sign}\{O_T\} |T|^2 d\mathrm{lips}}{\int |T|^2 d\mathrm{lips}} \,, \tag{1.4}$$

where $\int |T|^2 d\mathrm{lips}$ is proportional to the cross section $\sigma(e^+e^- \to \tilde{\chi}^0_k \tilde{\chi}^0_2 \to \tilde{\chi}^0_k \tilde{\chi}^0_1 \ell^+ \ell^-)$. It is essential to include the spin correlations between production and decay, because otherwise the numerator and hence the asymmetry A_T would vanish. This observable has the advantage that it is not necessary to reconstruct the momentum of the decaying neutralino. The asymmetry A_T, eq. (1.4), is odd under the naive time-reversal operation. By CPT it is a CP-sensitive asymmetry, if final-state interactions and finite-widths effects are unimportant. We will neglect these effects, because they are of higher order. By its definition, the asymmetry A_T is the difference of the number of events with the final lepton ℓ^+ above and below the plane spanned by $\vec{p}_{\ell^-} \times \vec{p}_{e^-}$, normalised by the sum of these events.

In this paper we will first present the complete analytic formulae at tree level for the cross section and for the asymmetry (1.4) of the processes (1.1) and (1.2), taking into account the full spin correlations between production and decay of the neutralino $\tilde{\chi}^0_2$ [16]. We will study the dependence of the asymmetry and the cross section on the phases ϕ_μ and ϕ_{M_1} and present detailed numerical results for an e^+e^- linear collider with $\sqrt{s} = 500\,\mathrm{GeV}$ and polarised beams. Analogous CP asymmetries in neutralino production and subsequent two-body decays have been studied in [17]. CP asymmetries based on triple products in decays of scalar fermions have been discussed in [18]. CP-odd observables involving the polarisation of the outgoing τ leptons from neutralinos decaying via two-body decays $\tilde{\chi}^0_2 \to \tilde{\tau}^\pm_1 \tau^\mp \to \tilde{\chi}^0_1 \tau^\pm \tau^\mp$ have been analysed in [19]. A systematic study of the impact of the supersymmetric phases on neutralino, chargino and selectron production at linear colliders has been performed in [12]. A Monte Carlo study of the asymmetry (1.4) for $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2 \to \tilde{\chi}^0_1\tilde{\chi}^0_1\ell^+\ell^-$, $l = e, \mu$, including SM backgrounds and detector effects has been given in [20], however, no analytic formulae for the cross section and the asymmetries have been given there. In [21] the impact of the phases ϕ_{A_τ}, ϕ_{A_t}, ϕ_{A_b}, ϕ_μ and ϕ_{M_1} on the two-body decays of the third generation sfermions has been analysed in detail. The influence of these phases on the polarisation of the outgoing fermions in third generation sfermion decays has been studied in [22].

In section 2 we shortly present the basics of the spin density matrix formalism. In section 3 we give the analytic expressions for the asymmetry A_T and the cross sections. Section 4 contains numerical results and discussions. In section 5 we present our conclusions. Appendices A and B contain some details of the formalism and of the analytic expressions.

2. Formalism

2.1 Lagrangian and couplings

The production process $e^+e^- \to \tilde{\chi}^0_i\tilde{\chi}^0_2$, $i = 1, \ldots, 4$, proceeds via Z^0 and $\tilde{e}_{L,R}$ exchange (figure 1). In the decay process $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1\ell^+\ell^-$, $\ell = e, \mu$, Z^0 and $\tilde{\ell}_{L,R}$ exchanges contribute (figure 2).

The interaction lagrangians for these processes are [23] (in our notation and conventions we follow [16])

$$\mathcal{L}_{Z^0\ell^+\ell^-} = -\frac{g}{\cos\Theta_W} Z_\mu \bar{\ell}\gamma^\mu \left[L_\ell P_L + R_\ell P_R\right]\ell\,, \tag{2.1}$$

$$\mathcal{L}_{Z^0\tilde{\chi}^0_m\tilde{\chi}^0_n} = \frac{1}{2}\frac{g}{\cos\Theta_W} Z_\mu \bar{\tilde{\chi}}^0_m \gamma^\mu [O''^L_{mn} P_L + O''^R_{mn} P_R]\tilde{\chi}^0_n\,, \tag{2.2}$$

$$\mathcal{L}_{\ell\tilde{\ell}\tilde{\chi}^0_k} = g f^L_{\ell k}\bar{\ell}P_R\tilde{\chi}^0_k\tilde{\ell}_L + g f^R_{\ell k}\bar{\ell}P_L\tilde{\chi}^0_k\tilde{\ell}_R + \mathrm{h.c.}\,, \tag{2.3}$$

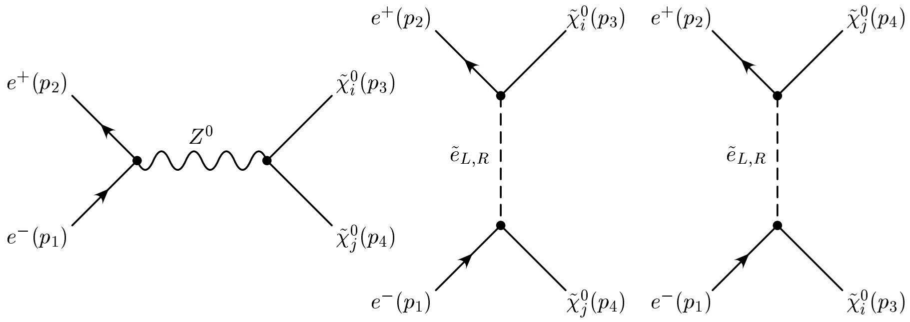

Figure 1: Feynman diagrams of the production process $e^+e^- \to \tilde{\chi}^0_i \tilde{\chi}^0_j$.

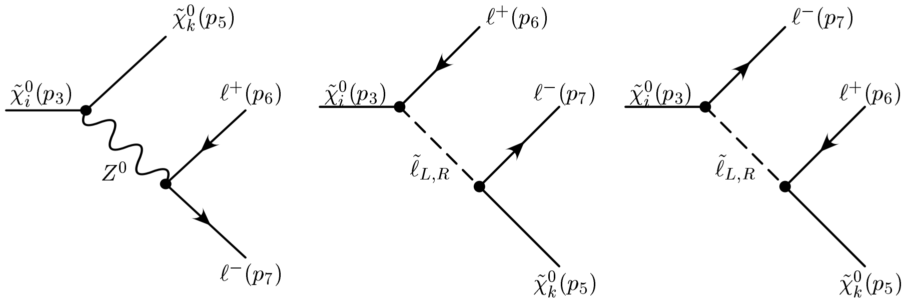

Figure 2: Feynman diagrams of the three-body decay $\tilde{\chi}^0_i \to \tilde{\chi}^0_k \ell^+ \ell^-$.

with $m, n, k = 1, \ldots, 4$ and the couplings

$$f^L_{\ell k} = -\sqrt{2}\left[\frac{1}{\cos\Theta_W}(T_{3\ell} - e_\ell \sin^2\Theta_W)N_{k2} + e_\ell \sin\Theta_W N_{k1}\right], \tag{2.4}$$

$$f^R_{\ell k} = -\sqrt{2} e_\ell \sin\Theta_W \left[\tan\Theta_W N^*_{k2} - N^*_{k1}\right], \tag{2.5}$$

$$O''^L_{mn} = -\frac{1}{2}(N_{m3}N^*_{n3} - N_{m4}N^*_{n4})\cos 2\beta - \frac{1}{2}(N_{m3}N^*_{n4} + N_{m4}N^*_{n3})\sin 2\beta, \tag{2.6}$$

$$O''^R_{mn} = -O''^{L*}_{mn}, \tag{2.7}$$

$$L_\ell = T_{3\ell} - e_\ell \sin^2\Theta_W, \quad R_\ell = -e_\ell \sin^2\Theta_W, \tag{2.8}$$

where $P_{L,R} = \frac{1}{2}(1 \mp \gamma_5)$, g is the weak coupling constant ($g = e/\cos\Theta_W$, $e > 0$). e_ℓ and $T_{3\ell}$ are the charge (in units of e) and the third component of the weak isospin of the fermion ℓ. Θ_W is the weak mixing angle and $\tan\beta = v_2/v_1$ is the ratio of the vacuum expectation values of the Higgs fields. The unitary (4×4)-matrix N_{mk} which diagonalises the complex symmetric neutralino mass matrix is given in the basis $(\tilde{\gamma}, \tilde{Z}, \tilde{H}^0_a, \tilde{H}^0_b)$ [4].

2.2 Amplitudes

From these interaction lagrangians the helicity amplitudes $T^{\lambda_i\lambda_j}_P(\alpha)$ and $T_{D,\lambda_i}(\alpha)$ for production and decay can be calculated. Here α denotes the production or decay channel and the respective exchanged particle. λ_i and λ_j are the helicities of the neutralinos $\tilde{\chi}^0_i$ and $\tilde{\chi}^0_j$. In $T_{D,\lambda_i}(\alpha)$ we suppress the label of the helicity of the final neutralino $\tilde{\chi}^0_k$. The amplitudes

$T_P^{\lambda_i\lambda_j}(\alpha)$ for the production process $e^-(p_1)e^+(p_2) \to \tilde{\chi}_i^0(p_3)\tilde{\chi}_j^0(p_4)$ (see figure 1) are

$$T_P^{\lambda_i\lambda_j} = T_P^{\lambda_i\lambda_j}(s,Z) + T_P^{\lambda_i\lambda_j}(t,\tilde{e}_L) + T_P^{\lambda_i\lambda_j}(t,\tilde{e}_R) + T_P^{\lambda_i\lambda_j}(u,\tilde{e}_L) + T_P^{\lambda_i\lambda_j}(u,\tilde{e}_R)\,, \tag{2.9}$$

where

$$\begin{aligned} T_P^{\lambda_i\lambda_j}(s,Z) &= \frac{g^2}{\cos^2\Theta_W}\Delta^s(Z)\bar{v}(p_2)\gamma^\mu(L_\ell P_L + R_\ell P_R)u(p_1)\times \\ &\quad \times \bar{u}(p_4,\lambda_j)\gamma_\mu(O''^L_{ji}P_L + O''^R_{ji}P_R)v(p_3,\lambda_i)\,, &(2.10)\\ T_P^{\lambda_i\lambda_j}(t,\tilde{e}_L) &= -g^2 f^L_{ei}f^{L*}_{ej}\Delta^t(\tilde{e}_L)\bar{v}(p_2)P_R v(p_3,\lambda_i)\bar{u}(p_4,\lambda_j)P_L u(p_1)\,, &(2.11)\\ T_P^{\lambda_i\lambda_j}(t,\tilde{e}_R) &= -g^2 f^R_{ei}f^{R*}_{ej}\Delta^t(\tilde{e}_R)\bar{v}(p_2)P_L v(p_3,\lambda_i)\bar{u}(p_4,\lambda_j)P_R u(p_1)\,, &(2.12)\\ T_P^{\lambda_i\lambda_j}(u,\tilde{e}_L) &= g^2 f^{L*}_{ei}f^{L}_{ej}\Delta^u(\tilde{e}_L)\bar{v}(p_2)P_R v(p_4,\lambda_j)\bar{u}(p_3,\lambda_i)P_L u(p_1)\,, &(2.13)\\ T_P^{\lambda_i\lambda_j}(u,\tilde{e}_R) &= g^2 f^{R*}_{ei}f^{R}_{ej}\Delta^u(\tilde{e}_R)\bar{v}(p_2)P_L v(p_4,\lambda_j)\bar{u}(p_3,\lambda_i)P_R u(p_1) &(2.14) \end{aligned}$$

with

$$\Delta^s(Z) = \frac{i}{s - m_Z^2 + i m_Z\Gamma_Z}\,, \tag{2.15}$$

$$\Delta^t(\tilde{e}_{L,R}) = \frac{i}{t - m^2_{\tilde{e}_{L,R}} + i m_{\tilde{e}_{L,R}}\Gamma_{\tilde{e}_{L,R}}}, \qquad \Delta^u(\tilde{e}_{L,R}) = \frac{i}{u - m^2_{\tilde{e}_{L,R}} + i m_{\tilde{e}_{L,R}}\Gamma_{\tilde{e}_{L,R}}} \tag{2.16}$$

and the Mandelstam variables

$$s = (p_1+p_2)^2\,, \qquad t = (p_1-p_4)^2\,, \qquad u = (p_1-p_3)^2\,. \tag{2.17}$$

$\Delta^s(Z)$, m_Z, Γ_Z, $\Delta^{t,u}(\tilde{e}_{L,R})$, $m_{\tilde{e}_{L,R}}$, $\Gamma_{\tilde{e}_{L,R}}$ denote the corresponding propagator, mass and width of the exchanged particle. The amplitudes $T_{D,\lambda_i}(\alpha)$ for the decay $\tilde{\chi}_i^0(p_3) \to \tilde{\chi}_k^0(p_5)\ell^+(p_6)\ell^-(p_7)$ (see figure 2) read

$$T_{D,\lambda_i} = T_{D,\lambda_i}(s_i,Z) + T_{D,\lambda_i}(t_i,\tilde{\ell}_L) + T_{D,\lambda_i}(t_i,\tilde{\ell}_R) + T_{D,\lambda_i}(u_i,\tilde{\ell}_L) + T_{D,\lambda_i}(u_i,\tilde{\ell}_R) \tag{2.18}$$

with

$$\begin{aligned} T_{D,\lambda_i}(s_i,Z) &= -\frac{g^2}{\cos^2\Theta_W}\Delta^{s_i}(Z)\bar{u}(p_7)\gamma^\mu(L_\ell P_L + R_\ell P_R)v(p_6)\times \\ &\quad \times \bar{u}(p_5)\gamma_\mu(O''^L_{ki}P_L + O''^R_{ki}P_R)u(p_3,\lambda_i)\,, &(2.19)\\ T_{D,\lambda_i}(t_i,\tilde{\ell}_L) &= -g^2 f^L_{\ell k}f^{L*}_{\ell i}\Delta^{t_i}(\tilde{\ell}_L)\bar{u}(p_7)P_R v(p_5)\bar{v}(p_3,\lambda_i)P_L v(p_6)\,, &(2.20)\\ T_{D,\lambda_i}(t_i,\tilde{\ell}_R) &= -g^2 f^R_{\ell k}f^{R*}_{\ell i}\Delta^{t_i}(\tilde{\ell}_R)\bar{u}(p_7)P_L v(p_5)\bar{v}(p_3,\lambda_i)P_R v(p_6)\,, &(2.21)\\ T_{D,\lambda_i}(u_i,\tilde{\ell}_L) &= +g^2 f^L_{\ell i}f^{L*}_{\ell k}\Delta^{u_i}(\tilde{\ell}_L)\bar{u}(p_7)P_R u(p_3,\lambda_i)\bar{u}(p_5)P_L v(p_6)\,, &(2.22)\\ T_{D,\lambda_i}(u_i,\tilde{\ell}_R) &= +g^2 f^R_{\ell i}f^{R*}_{\ell k}\Delta^{u_i}(\tilde{\ell}_R)\bar{u}(p_7)P_L u(p_3,\lambda_i)\bar{u}(p_5)P_R v(p_6) &(2.23) \end{aligned}$$

and the kinematic variables

$$s_i = (p_6+p_7)^2\,, \qquad t_i = (p_3-p_6)^2\,, \qquad u_i = (p_3-p_7)^2\,. \tag{2.24}$$

The amplitude for the whole process $e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0 \to \tilde{\chi}_1^0\ell^+\ell^-\tilde{\chi}_j^0$, with $\tilde{\chi}_i^0$ decaying, can be written as

$$T = \Delta(\tilde{\chi}_i^0)\sum_{\lambda_i} T_P^{\lambda_i\lambda_j} T_{D,\lambda_i}\,, \tag{2.25}$$

where $\Delta(\tilde{\chi}_i^0) = 1/[p_3^2 - m_i^2 + im_i\Gamma_i]$, m_i, Γ_i are the propagator, mass and width of the decaying neutralino $\tilde{\chi}_i^0$.

2.3 Cross section

Following the formalism of [16, 24], the amplitude squared $|T|^2$ of the combined processes of production $e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0$ and decay $\tilde{\chi}_i^0 \to \tilde{\chi}_1^0\ell^+\ell^-$ of the neutralino $\tilde{\chi}_i^0$, with neutralino $\tilde{\chi}_j^0$ being unobserved, can be written as

$$|T|^2 = 4|\Delta(\tilde{\chi}_i^0)|^2\left\{P(\tilde{\chi}_i^0\tilde{\chi}_j^0)D(\tilde{\chi}_i^0) + \sum_{a=1}^{3}\Sigma_P^a(\tilde{\chi}_i^0)\Sigma_D^a(\tilde{\chi}_i^0)\right\}, \tag{2.26}$$

where $a = 1,2,3$ refers to the polarisation state of the neutralino $\tilde{\chi}_i^0$, which is described by the polarisation vectors $s^a(\tilde{\chi}_i^0)$ given in appendix A. $P(\tilde{\chi}_i^0\tilde{\chi}_j^0)$ and $D(\tilde{\chi}_i^0)$ are the terms of production and decay independent of the polarisation of the decaying neutralino, whereas $\Sigma_P^a(\tilde{\chi}_i^0)$ and $\Sigma_D^a(\tilde{\chi}_i^0)$ are the terms containing the spin correlations between production and decay. According to our choice of the polarisation vectors $s^a(\tilde{\chi}_i^0)$, eqs. (A.6)–(A.8), Σ_P^3/P is the longitudinal polarisation, Σ_P^1/P is the transverse polarisation in the production plane and Σ_P^2/P is the polarisation perpendicular to the production plane of the neutralino $\tilde{\chi}_i^0$. The explicit expressions for these contributions are given in appendix B. The complete expressions for the amplitude squared including spin-spin correlations for the case when both neutralinos are decaying and for longitudinally polarised $e^\pm$ beams are given in [16]. The differential cross section in the laboratory system is

$$d\sigma = \frac{1}{8E_b^2}|T|^2(2\pi)^4\delta^4\left(p_1 + p_2 - \sum_{i=4}^{7}p_i\right)d\mathrm{lips}(p_3\cdots p_7)\,, \tag{2.27}$$

where E_b is the beam energy and $d\mathrm{lips}(p_3\cdots p_7)$ is the Lorentz invariant phase space element. Integrating over all angles gives the cross section for the combined process of production and decay

$$\begin{aligned}\sigma &= \sigma(e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0)\cdot BR(\tilde{\chi}_i^0 \to \tilde{\chi}_1^0\ell^+\ell^-)\\ &= \frac{1}{2E_b^2}\int P(\tilde{\chi}_i^0\tilde{\chi}_j^0)D(\tilde{\chi}_i^0)|\Delta(\tilde{\chi}_i^0)|^2(2\pi)^4\delta^4\left(p_1 + p_2 - \sum_{i=4}^{7}p_i\right)d\mathrm{lips}(p_3\cdots p_7)\,. \end{aligned}\tag{2.28}$$

3. Triple product and CP asymmetry

In this section we will derive the analytic formulae for the cross section $\sigma(e^+e^- \to \tilde{\chi}_i^0\tilde{\chi}_j^0 \to \tilde{\chi}_j^0\tilde{\chi}_1^0\ell^+\ell^-)$ and for the asymmetry A_T, eq. (1.4). As can be seen from the numerator of A_T, for this purpose we have to identify those terms in $|T|^2$, eq. (2.26), which contain a triple product of the form eq. (1.3). Triple products follow from expressions $i\epsilon_{\mu\nu\rho\sigma}k^\mu p^\nu q^\rho s^\sigma$ in the terms $\Sigma^a_P(\tilde{\chi}^0_i)$ and $\Sigma^a_D(\tilde{\chi}^0_i)$, where k, p, q, s are 4-momenta and spins of the particles involved (see appendix B, eqs. (B.11)–(B.19) and (B.28)–(B.36)). The expressions $i\epsilon_{\mu\nu\rho\sigma}k^\mu p^\nu q^\rho s^\sigma$ are imaginary and when multiplied by the imaginary parts of the respective couplings they yield the terms which contribute to the numerator of A_T, eq. (1.4). Hence only the second term of eq. (2.26), which is due to the spin correlations, contributes to the numerator of the asymmetry A_T. It is convenient to split $\Sigma^a_P(\tilde{\chi}^0_i)$ and $\Sigma^a_D(\tilde{\chi}^0_i)$ into T-odd terms $\Sigma^{a,\mathrm{O}}_P(\tilde{\chi}^0_i)$ and $\Sigma^{a,\mathrm{O}}_D(\tilde{\chi}^0_i)$ containing the respective triple product, and T-even terms $\Sigma^{a,\mathrm{E}}_P(\tilde{\chi}^0_i)$ and $\Sigma^{a,\mathrm{E}}_D(\tilde{\chi}^0_i)$ without triple products:

$$\Sigma^a_P(\tilde{\chi}^0_i) = \Sigma^{a,\mathrm{O}}_P(\tilde{\chi}^0_i) + \Sigma^{a,\mathrm{E}}_P(\tilde{\chi}^0_i)\,, \qquad \Sigma^a_D(\tilde{\chi}^0_i) = \Sigma^{a,\mathrm{O}}_D(\tilde{\chi}^0_i) + \Sigma^{a,\mathrm{E}}_D(\tilde{\chi}^0_i)\,. \tag{3.1}$$

Then the terms of $|T|^2$, eq. (2.26), which contribute to the numerator of A_T are

$$|T|^2 \supset 4|\Delta(\tilde{\chi}^0_i)|^2 \sum_{a=1}^{3} \left[\Sigma^{a,\mathrm{O}}_P(\tilde{\chi}^0_i)\Sigma^{a,\mathrm{E}}_D(\tilde{\chi}^0_i) + \Sigma^{a,\mathrm{E}}_P(\tilde{\chi}^0_i)\Sigma^{a,\mathrm{O}}_D(\tilde{\chi}^0_i)\right], \tag{3.2}$$

where the first (second) term is sensitive to the CP phases in the production (decay) of the neutralino $\tilde{\chi}^0_i$. In the following we derive explicitly the T-odd contributions to the spin density matrices of production and decay.

3.1 T-odd terms of the production density matrix

As can be seen from eq. (B.19) in appendix B, in $\Sigma^a_P(\tilde{\chi}^0_i)$ the expressions f^a_4 vanish for $a = 1, 3$, because they do not contain three linearly independent vectors, hence $\Sigma^{1,\mathrm{O}}_P(\tilde{\chi}^0_i) = \Sigma^{3,\mathrm{O}}_P(\tilde{\chi}^0_i) = 0$. Only for $a = 2$ we get a T-odd contribution to the production density matrix $\Sigma^{2,\mathrm{O}}_P(\tilde{\chi}^0_i)$, which is related to the polarisation of the neutralino $\tilde{\chi}^0_i$ perpendicular to the production plane (see the expression for $s^2(\tilde{\chi}^0_i)$, eq. (A.7) in appendix A). We obtain from eqs.(B.12)–(B.15) for $a = 2$

$$\Sigma^{2,\mathrm{O}}_P(\tilde{\chi}^0_i) = \Sigma^{2,\mathrm{O}}_P(ZZ) + \Sigma^{2,\mathrm{O}}_P(Z\tilde{e}_L) + \Sigma^{2,\mathrm{O}}_P(Z\tilde{e}_R) + \Sigma^{2,\mathrm{O}}_P(\tilde{e}_L\tilde{e}_L) + \Sigma^{2,\mathrm{O}}_P(\tilde{e}_R\tilde{e}_R) \tag{3.3}$$

with

$$\begin{aligned} \Sigma^{2,\mathrm{O}}_P(ZZ) &= \frac{g^4}{\cos^4\Theta_W}|\Delta^s(Z)|^2\Big(R^2_\ell(1+P_{e^-})(1-P_{e^+}) - L^2_\ell(1-P_{e^-})(1+P_{e^+})\Big)\times \\ &\quad\times\left[2(\mathrm{Re}O''^L_{ij})(\mathrm{Im}O''^L_{ij})if^2_4\right], \end{aligned} \tag{3.4}$$

$$\begin{aligned} \Sigma^{2,\mathrm{O}}_P(Z\tilde{e}_L) &= \frac{g^4}{2\cos^2\Theta_W}L_\ell(1-P_{e^-})(1+P_{e^+})\times \\ &\quad\times\mathrm{Re}\left\{\Delta^s(Z)\left[f^{L*}_{\ell i}f^L_{\ell j}O''^{L*}_{ij}\Delta^{t*}(\tilde{e}_L) - f^L_{\ell i}f^{L*}_{\ell j}O''^L_{ij}\Delta^{u*}(\tilde{e}_L)\right]f^2_4\right\}, \end{aligned} \tag{3.5}$$

$$\Sigma^{2,\mathrm{O}}_P(\tilde{e}_L\tilde{e}_L) = \frac{g^4}{4}(1-P_{e^-})(1+P_{e^+})\mathrm{Re}\left\{\Delta^u(\tilde{e}_L)\Delta^{t*}(\tilde{e}_L)(f^{L*}_{\ell i})^2(f^L_{\ell j})^2f^2_4\right\}, \tag{3.6}$$

$$\begin{aligned} \Sigma^{2,\mathrm{O}}_P(Z\tilde{e}_R) &= \frac{g^4}{2\cos^2\Theta_W}R_\ell(1+P_{e^-})(1-P_{e^+})\times \\ &\quad\times\mathrm{Re}\left\{\Delta^s(Z)\left[f^{R*}_{\ell i}f^R_{\ell j}O''^{R*}_{ij}\Delta^{t*}(\tilde{e}_R) - f^R_{\ell i}f^{R*}_{\ell j}O''^R_{ij}\Delta^{u*}(\tilde{e}_R)\right]f^2_4\right\}, \end{aligned} \tag{3.7}$$

$$\Sigma^{2,\mathrm{O}}_P(\tilde{e}_R\tilde{e}_R) = \frac{g^4}{4}(1+P_{e^-})(1-P_{e^+})\mathrm{Re}\left\{\Delta^u(\tilde{e}_R)\Delta^{t*}(\tilde{e}_R)(f^{R*}_{\ell i})^2(f^R_{\ell j})^2f^2_4\right\}, \tag{3.8}$$

where P_{e^-} and P_{e^+} denotes the degree of longitudinal polarisation of the electron beam and positron beam, respectively. The function f_4^a, eq. (B.19), for $a=2$ reads

$$f_4^2 = i m_j \epsilon_{\mu\nu\rho\sigma} p_2^\mu p_1^\nu s^{2\rho} p_3^\sigma \,. \tag{3.9}$$

Note that f_4^2 is purely imaginary. When inserted, for example, into the expression for $\Sigma_P^{2,\mathrm{O}}(Z\tilde{e}_L)$, eq. (3.5), f_4^2 is multiplied by the factor $i \cdot Im\{f_{\ell i}^L f_{\ell j}^{L*} O_{ij}^{''L}\}$, which is non-vanishing only if the couplings are complex. Hence it gives a CP-sensitive contribution to the asymmetry A_T, which depends on the phases ϕ_μ and ϕ_{M_1}. Analogous contributions come from the other terms in $\Sigma_P^{2,\mathrm{O}}$, eqs. (3.4),(3.6)–(3.8). We have to multiply $\Sigma_P^{2,\mathrm{O}}$ in eq. (3.2) by $\Sigma_D^{2,\mathrm{E}}$, for which we obtain from eqs. (B.28)–(B.32), appendix B,

$$\Sigma_D^{2,\mathrm{E}}(\tilde{\chi}_i^0) = \Sigma_D^{2,\mathrm{E}}(ZZ) + \Sigma_D^{2,\mathrm{E}}(Z\tilde{\ell}_L) + \Sigma_D^{2,\mathrm{E}}(Z\tilde{\ell}_R) + \Sigma_D^{2,\mathrm{E}}(\tilde{\ell}_L\tilde{\ell}_L) + \Sigma_D^{2,\mathrm{E}}(\tilde{\ell}_R\tilde{\ell}_R) \tag{3.10}$$

with

$$\begin{aligned}
\Sigma_D^{2,\mathrm{E}}(ZZ) &= 8\frac{g^4}{\cos^4\Theta_W}|\Delta^{s_i}(Z)|^2(R_\ell^2 - L_\ell^2)\times \\
&\quad\times\left[-[(\mathrm{Re}O_{ki}^{''L})^2 - (\mathrm{Im}O_{ki}^{''L})^2]g_3^2 + |O_{ki}^{''L}|^2(g_1^2 - g_2^2)\right], \qquad (3.11)\\
\Sigma_D^{2,\mathrm{E}}(Z\tilde{\ell}_L) &= \frac{4g^4}{\cos^2\Theta_W}L_\ell\mathrm{Re}\Big\{\Delta^{s_i}(Z)\Big[f_{\ell i}^L f_{\ell k}^{L*}\Delta^{t_i *}(\tilde{\ell}_L)\left(-2O_{ki}^{''L}g_1^2 + O_{ki}^{''L*}g_3^2\right) + \\
&\quad + f_{\ell i}^{L*} f_{\ell k}^L \Delta^{u_i *}(\tilde{\ell}_L)\left(2O_{ki}^{''L*}g_2^2 + O_{ki}^{''L}g_3^2\right)\Big]\Big\}, \qquad (3.12)\\
\Sigma_D^{2,\mathrm{E}}(\tilde{\ell}_L\tilde{\ell}_L) &= 2g^4\Big[|f_{\ell i}^L|^2|f_{\ell k}^L|^2[|\Delta^{u_i}(\tilde{\ell}_L)|^2 g_2^2 - |\Delta^{t_i}(\tilde{\ell}_L)|^2 g_1^2] + \\
&\quad + \mathrm{Re}\left\{(f_{\ell i}^{L*})^2(f_{\ell k}^L)^2\Delta^{t_i}(\tilde{\ell}_L)\Delta^{u_i *}(\tilde{\ell}_L)g_3^2\right\}\Big], \qquad (3.13)\\
\Sigma_D^{2,\mathrm{E}}(Z\tilde{\ell}_R) &= \frac{4g^4}{\cos^2\Theta_W}R_\ell\mathrm{Re}\Big\{\Delta^{s_i}(Z)\Big[f_{\ell i}^R f_{\ell k}^{R*}\Delta^{t_i *}(\tilde{\ell}_R)\left(2O_{ki}^{''R}g_1^2 - O_{ki}^{''R*}g_3^2\right) + \\
&\quad + f_{\ell i}^{R*} f_{\ell k}^R \Delta^{u_i *}(\tilde{\ell}_R)\left(-2O_{ki}^{''R*}g_2^2 - O_{ki}^{''R}g_3^2\right)\Big]\Big\}, \qquad (3.14)\\
\Sigma_D^{2,\mathrm{E}}(\tilde{\ell}_R\tilde{\ell}_R) &= 2g^4\Big[|f_{\ell i}^R|^2|f_{\ell k}^R|^2[-|\Delta^{u_i}(\tilde{\ell}_R)|^2 g_2^2 + |\Delta^{t_i}(\tilde{\ell}_R)|^2 g_1^2] - \\
&\quad - \mathrm{Re}\left\{(f_{\ell i}^{R*})^2(f_{\ell k}^R)^2\Delta^{t_i}(\tilde{\ell}_R)\Delta^{u_i *}(\tilde{\ell}_R)g_3^2\right\}\Big], \qquad (3.15)
\end{aligned}$$

where

$$\begin{aligned}
g_1^2 &= m_i(p_5p_7)(p_6s^2)\,, &(3.16)\\
g_2^2 &= m_i(p_5p_6)(p_7s^2)\,, &(3.17)\\
g_3^2 &= m_k[(p_3p_6)(p_7s^2) - (p_3p_7)(p_6s^2)]\,. &(3.18)
\end{aligned}$$

The kinematic functions g_1^2, g_2^2, g_3^2 are real. When multiplied by the purely imaginary f_4^2, eq. (3.9), this leads to triple products sensitive to the CP phases in the production process, which in the laboratory system read:

$$g_1^2 \cdot f_4^2 = i2E_b m_i m_j (p_5 p_7)\vec{p}_6(\vec{p}_1 \times \vec{p}_3)\,, \tag{3.19}$$
$$g_2^2 \cdot f_4^2 = i2E_b m_i m_j (p_5 p_6)\vec{p}_7(\vec{p}_1 \times \vec{p}_3)\,, \tag{3.20}$$
$$g_3^2 \cdot f_4^2 = i2E_b m_j m_k \{(p_3 p_6)\vec{p}_7(\vec{p}_1 \times \vec{p}_3) - (p_3 p_7)\vec{p}_6(\vec{p}_1 \times \vec{p}_3)\}\,. \tag{3.21}$$

As outlined above, these expressions will be multiplied in eqs. (3.4)–(3.8) by the factors $i \cdot \mathrm{Im}\{f_{\ell i}^L f_{\ell j}^{L*} O_{ij}^{''L}\}$ etc. and contribute to the first term of eq. (3.2) and, hence, to the numerator of the asymmetry A_T, eq. (1.4).

3.2 T-odd terms of the decay density matrix

The second term in eq. (3.2) is sensitive to CP violation in the neutralino decay process (1.2). The expressions g_4^a, eq. (B.36), in $\Sigma_D^{a,\mathrm{O}}(\tilde{\chi}_i^0)$ contain three linearly independent vectors, however, $\Sigma_P^{2,\mathrm{E}}(\tilde{\chi}_i^0)$ (eq. (3.29)) vanishes because $s^2(\tilde{\chi}_i^0)$ is perpendicular to the scattering plane. Therefore, only the terms $\Sigma_P^{a,\mathrm{E}}(\tilde{\chi}_i^0)$ for $a = 1, 3$ have to be taken into account. The CP-sensitive terms of the decay density matrix following from eqs. (B.28)–(B.32) for $a = 1, 3$, are

$$\Sigma_D^{a,\mathrm{O}}(\tilde{\chi}_i^0) = \Sigma_D^{a,\mathrm{O}}(ZZ) + \Sigma_D^{a,\mathrm{O}}(Z\tilde{\ell}_L) + \Sigma_D^{a,\mathrm{O}}(Z\tilde{\ell}_R) + \Sigma_D^{a,\mathrm{O}}(\tilde{\ell}_L\tilde{\ell}_L) + \Sigma_D^{a,\mathrm{O}}(\tilde{\ell}_R\tilde{\ell}_R) \tag{3.22}$$

with

$$\Sigma_D^{a,\mathrm{O}}(ZZ) = 8\frac{g^4}{\cos^4\Theta_W}|\Delta^{s_i}(Z)|^2(L_\ell^2 - R_\ell^2)\left[2\mathrm{Re}(O_{ki}^{''L})\mathrm{Im}(O_{ki}^{''L})ig_4^a\right], \tag{3.23}$$

$$\Sigma_D^{a,\mathrm{O}}(Z\tilde{\ell}_L) = \frac{4g^4}{\cos^2\Theta_W}L_\ell\mathrm{Re}\Big\{\Delta^{s_i}(Z)\Big[-f_{\ell i}^L f_{\ell k}^{L*} O_{ki}^{''L*}\Delta^{t_i *}(\tilde{\ell}_L) + f_{\ell i}^{L*} f_{\ell k}^L O_{ki}^{''L}\Delta^{u_i *}(\tilde{\ell}_L)\Big]g_4^a\Big\}, \tag{3.24}$$

$$\Sigma_D^{a,\mathrm{O}}(\tilde{\ell}_L\tilde{\ell}_L) = 2g^4\mathrm{Re}\left\{(f_{\ell i}^{L*})^2(f_{\ell k}^L)^2\Delta^{t_i}(\tilde{\ell}_L)\Delta^{u_i *}(\tilde{\ell}_L)g_4^a\right\}, \tag{3.25}$$

$$\Sigma_D^{a,\mathrm{O}}(Z\tilde{\ell}_R) = \frac{4g^4}{\cos^2\Theta_W}R_\ell\mathrm{Re}\Big\{\Delta^{s_i}(Z)\Big[-f_{\ell i}^R f_{\ell k}^{R*} O_{ki}^{''R*}\Delta^{t_i *}(\tilde{\ell}_R) + f_{\ell i}^{R*} f_{\ell k}^R O_{ki}^{''L}\Delta^{u_i *}(\tilde{\ell}_R)\Big]g_4^a\Big\}, \tag{3.26}$$

$$\Sigma_D^{a,\mathrm{O}}(\tilde{\ell}_R\tilde{\ell}_R) = 2g^4\mathrm{Re}\left\{(f_{\ell i}^{R*})^2(f_{\ell k}^R)^2\Delta^{t_i}(\tilde{\ell}_R)\Delta^{u_i *}(\tilde{\ell}_R)g_4^a\right\}, \tag{3.27}$$

where for $a = 1, 3$ we have

$$g_4^a = i m_k \epsilon_{\mu\nu\rho\sigma} s^{a\mu} p_3^\nu p_7^\rho p_6^\sigma\,. \tag{3.28}$$

Now g_4^a, $a = 1, 3$, is purely imaginary. When inserted, for example, in eq. (3.24) it is multiplied by the factor $i \cdot Im\{f_{\ell i}^L f_{\ell k}^{L*} O_{ki}^{''L*}\}$, which depends on the phases ϕ_μ and ϕ_{M_1} and contributes to $\Sigma_D^{a,\mathrm{O}}$. Analogous contributions follow from eqs. (3.23), (3.25)–(3.27). The corresponding T-even terms of the production density matrix also entering in eq. (3.2) are obtained from eqs. (B.12)–(B.15),

$$\Sigma_P^{a,\mathrm{E}}(\tilde{\chi}_i^0) = \Sigma_P^{a,\mathrm{E}}(ZZ) + \Sigma_P^{a,\mathrm{E}}(Z\tilde{e}_L) + \Sigma_P^{a,\mathrm{E}}(Z\tilde{e}_R) + \Sigma_P^{a,\mathrm{E}}(\tilde{e}_L\tilde{e}_L) + \Sigma_P^{a,\mathrm{E}}(\tilde{e}_R\tilde{e}_R)\,, \tag{3.29}$$

where

$$\begin{aligned}
\Sigma_P^{a,\mathrm{E}}(ZZ) &= \frac{g^4}{\cos^4\Theta_W}|\Delta^s(Z)|^2\Big(R_\ell^2(1+P_{e^-})(1-P_{e^+}) - L_\ell^2(1-P_{e^-})(1+P_{e^+})\Big)\times \\
&\quad\times\Big[|O_{ij}^{''L}|^2(f_2^a - f_1^a) - [(\mathrm{Re}O_{ij}^{''L})^2 - (\mathrm{Im}O_{ij}^{''L})^2]f_3^a\Big], \qquad (3.30)\\
\Sigma_P^{a,\mathrm{E}}(Z\tilde{e}_L) &= \frac{g^4}{2\cos^2\Theta_W}L_\ell(1-P_{e^-})(1+P_{e^+})\times \\
&\quad\times\mathrm{Re}\Big\{\Delta^s(Z)\Big[2f_{\ell i}^L f_{\ell j}^{L*}O_{ij}^{''L*}\Delta^{u*}(\tilde{e}_L)f_1^a - 2f_{\ell i}^{L*}f_{\ell j}^L O_{ij}^{''L}\Delta^{t*}(\tilde{e}_L)f_2^a + \\
&\qquad + \Big[f_{\ell i}^L f_{\ell j}^{L*}O_{ij}^{''L}\Delta^{u*}(\tilde{e}_L) + f_{\ell i}^{L*}f_{\ell j}^L O_{ij}^{''L*}\Delta^{t*}(\tilde{e}_L)\Big]\,f_3^a\Big]\Big\}, \qquad (3.31)\\
\Sigma_P^{a,\mathrm{E}}(\tilde{e}_L\tilde{e}_L) &= \frac{g^4}{4}(1-P_{e^-})(1+P_{e^+})\Big[|f_{\ell i}^L|^2|f_{\ell j}^L|^2(|\Delta^u(\tilde{e}_L)|^2 f_1^a - |\Delta^t(\tilde{e}_L)|^2 f_2^a) + \\
&\qquad + \mathrm{Re}\{(f_{\ell i}^{L*})^2(f_{\ell j}^L)^2\Delta^u(\tilde{e}_L)\Delta^{t*}(\tilde{e}_L)f_3^a\}\Big], \qquad (3.32)\\
\Sigma_P^{a,\mathrm{E}}(Z\tilde{e}_R) &= \frac{g^4}{2\cos^2\Theta_W}R_\ell(1+P_{e^-})(1-P_{e^+})\times \\
&\quad\times\mathrm{Re}\Big\{\Delta^s(Z)\Big[-2f_{\ell i}^R f_{\ell j}^{R*}O_{ij}^{''R*}\Delta^{u*}(\tilde{e}_R)f_1^a + 2f_{\ell i}^{R*}f_{\ell j}^R O_{ij}^{''R}\Delta^{t*}(\tilde{e}_R)f_2^a - \\
&\qquad - \Big[f_{\ell i}^R f_{\ell j}^{R*}O_{ij}^{''R}\Delta^{u*}(\tilde{e}_R) + f_{\ell i}^{R*}f_{\ell j}^R O_{ij}^{''R*}\Delta^{t*}(\tilde{e}_R)\Big]\,f_3^a\Big]\Big\}, \qquad (3.33)\\
\Sigma_P^{a,\mathrm{E}}(\tilde{e}_R\tilde{e}_R) &= \frac{g^4}{4}(1+P_{e^-})(1-P_{e^+})\Big[|f_{\ell i}^R|^2|f_{\ell j}^R|^2\left(-|\Delta^u(\tilde{e}_R)|^2 f_1^a + |\Delta^t(\tilde{e}_R)|^2 f_2^a\right) - \\
&\qquad - \mathrm{Re}\{(f_{\ell i}^{R*})^2(f_{\ell j}^R)^2\Delta^u(\tilde{e}_R)\Delta^{t*}(\tilde{e}_R)f_3^a\}\Big], \qquad (3.34)
\end{aligned}$$

where

$$\begin{aligned}
f_1^a &= m_i(p_2p_4)(p_1s^a), &(3.35)\\
f_2^a &= m_i(p_1p_4)(p_2s^a), &(3.36)\\
f_3^a &= m_j[(p_1p_3)(p_2s^a) - (p_2p_3)(p_1s^a)]\,. &(3.37)
\end{aligned}$$

The triple products sensitive to the CP phases in the decay read in the laboratory system:

$$\begin{aligned}
\sum_{a=1,3} f_1^a\cdot g_4^a &= im_im_k(p_2p_4)\Big\{-E_b\vec{p}_5(\vec{p}_7\times\vec{p}_6) - E_7\vec{p}_5(\vec{p}_6\times\vec{p}_1) + \\
&\qquad + E_6\vec{p}_5(\vec{p}_7\times\vec{p}_1) + E_5\vec{p}_1(\vec{p}_7\times\vec{p}_6)\Big\}, \qquad (3.38)
\end{aligned}$$

$$\sum_{a=1,3} f_2^a \cdot g_4^a = i m_i m_k (p_1 p_4)\Big\{ -E_b \vec{p}_5(\vec{p}_7 \times \vec{p}_6) + E_7 \vec{p}_5(\vec{p}_6 \times \vec{p}_1) - \\ - E_6 \vec{p}_5(\vec{p}_7 \times \vec{p}_1) - E_5 \vec{p}_1(\vec{p}_7 \times \vec{p}_6)\Big\}, \quad (3.39)$$

$$\sum_{a=1,3} f_3^a \cdot g_4^a = i m_j m_k \Big\{ [(p_2 p_3) - (p_1 p_3)] E_b \vec{p}_5(\vec{p}_7 \times \vec{p}_6) + \\ + [(p_2 p_3) + (p_1 p_3)] \\ \Big[E_7 \vec{p}_5(\vec{p}_6 \times \vec{p}_1) - E_6 \vec{p}_5(\vec{p}_7 \times \vec{p}_1) - E_5 \vec{p}_1(\vec{p}_7 \times \vec{p}_6)\Big]\Big\}. \quad (3.40)$$

As already mentioned, these quantities will be multiplied in eqs. (3.23)–(3.27) by the factor $i \cdot Im\{f^L_{\ell i} f^{L*}_{\ell k} O''^{L*}_{ki}\}$ etc. and contribute to the second term of eq. (3.2).

We take into account only the contributions to the T-odd asymmetry A_T which stem from the complex couplings and neglect the contributions from the widths of Z, $\tilde{\ell}_L$ and $\tilde{\ell}_R$ in the propagators eqs. (2.15), (2.16), because they are of higher order.

4. Numerical results

In this section we analyse numerically the CP asymmetry A_T, eq. (1.4), and the cross section for reactions (1.1) and (1.2), at an e^+e^- linear collider with $\sqrt{s} = 500\,\mathrm{GeV}$ and longitudinally polarised $e^\pm$ beams. In particular, we investigate the dependence on the phases of the complex parameters $M_1 = |M_1|e^{i\phi_{M_1}}$ and $\mu = |\mu|e^{i\phi_\mu}$. For our numerical analysis we choose two scenarios A and B, such that $m_{\tilde{\chi}^0_2} < m_{\tilde{\chi}^0_1} + m_{Z^0}$ and $m_{\tilde{\chi}^0_2} < m_{\tilde{\ell}_{L,R}}$ to prevent two-body decays of $\tilde{\chi}^0_2$. The SUSY parameters and the masses of $\tilde{\chi}^0_i$, $i = 1,\ldots,4$, and $\tilde{\ell}_L$, $\tilde{\ell}_R$ are given in Table 1. The decay widths of the neutralinos have been computed with help of the program SPheno [25].

In scenario A the neutralinos $\tilde{\chi}^0_1$, $\tilde{\chi}^0_2$, $\tilde{\chi}^0_3$ are mainly $\tilde{B}$-higgsino, higgsino-$\tilde{B}$-$\tilde{W}^3$, higgsino-$\tilde{B}$ mixtures, respectively, where the $\tilde{B}$ component of the $\tilde{\chi}^0_2$ has a relatively strong ϕ_{M_1} dependence. In figure 3 we show the total decay width $\Gamma(\tilde{\chi}^0_2)$ and the branching ratio $BR(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \ell^+\ell^-)$, $\ell = e$ or μ in scenario A as a function of ϕ_{M_1} for $\phi_\mu = 0$. As can be seen, the CP-even quantities $\Gamma(\tilde{\chi}^0_2)$ and $BR(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \ell^+\ell^-)$ depend quite strongly on ϕ_{M_1}. $\Gamma(\tilde{\chi}^0_2)$ varies between 34 keV for $\phi_{M_1} = 0.25\pi$, 1.75π and 4.2 keV for $\phi_{M_1} = \pi$, and $BR(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \ell^+\ell^-)$ between 6.4 % for $\phi_{M_1} = 0, 2\pi$ and 2.6 % for $\phi_{M_1} = \pi$. The parameters M_2, $|\mu|$ and $\tan\beta$ in scenario B are chosen like in the scenario SPS1a in [26], with $\tilde{\chi}^0_1$, $\tilde{\chi}^0_2$, $\tilde{\chi}^0_3$ being mainly $\tilde{B}$-, $\tilde{W}^3$-, higgsino-like, respectively. In this scenario $\Gamma(\tilde{\chi}^0_2)$ varies between 80 keV for $\phi_{M_1} = 0$, 2π and 9.9 keV for $\phi_{M_1} = \pi$, and $B(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \ell^+\ell^-)$ between 1.9 % for $\phi_{M_1} = 0.4\pi$, 1.6π and 2.9 % for $\phi_{M_1} = \pi$. In both scenarios we take for the slepton masses $m_{\tilde{\ell}_L} = 267.6\,\mathrm{GeV}$, $m_{\tilde{\ell}_R} = 224.4\,\mathrm{GeV}$ and for the squark masses $m_{\tilde{u}_R} = 597.6\,\mathrm{GeV}$, $m_{\tilde{u}_L} = 599.0\,\mathrm{GeV}$ and $m_{\tilde{d}_R} = 600.5\,\mathrm{GeV}$, $m_{\tilde{d}_L} = 602.9\,\mathrm{GeV}$ for the first and second generation, and $m_{\tilde{b}_1} = 587.3\,\mathrm{GeV}$, $m_{\tilde{b}_2} = 615.7\,\mathrm{GeV}$ ($A_b = 1000\,\mathrm{GeV}$) for the bottom squarks. In most of the examples we take values for the phases ϕ_μ and ϕ_{M_1} which are in agreement with the constraints from the electron and neutron EDMs. In order to

JHEP08(2004)038

Scenario	A		B	
$\lvert M_1 \rvert$	150		90	
M_2	300		192.7	
$\lvert\mu\rvert$	200		352.4	
$\tan\beta$	10		10	
$m_{\tilde{\ell}_L}$	267.6		267.6	
$m_{\tilde{\ell}_R}$	224.4		224.4	
(ϕ_{M_1}, ϕ_μ)	$(0.5\pi, 0)$	$(0.5\pi, 0.5\pi)$	$(0.5\pi, 0)$	$(0.5\pi, 0.5\pi)$
$m_{\tilde{\chi}^0_1}$	135.4	139.9	88.5	89.7
$m_{\tilde{\chi}^0_2}$	182.2	182.8	176.2	180.3
$m_{\tilde{\chi}^0_3}$	213.3	214.9	359.4	360.7
$m_{\tilde{\chi}^0_4}$	334.8	331.6	377.1	373.6

Table 1: Input parameters $|M_1|$, M_2, $|\mu|$, $\tan\beta$, $m_{\tilde{\ell}_L}$ and $m_{\tilde{\ell}_R}$ and the resulting masses $m_{\tilde{\chi}^0_i}$, $i = 1, \ldots, 4$, for $(\phi_{M_1}, \phi_\mu) = (0.5\pi, 0)$ and $(0.5\pi, 0.5\pi)$. The parameters M_2, $|\mu|$ and $\tan\beta$ in scenario B are chosen according to the scenario SPS1a in [26]. All masses are given in GeV.

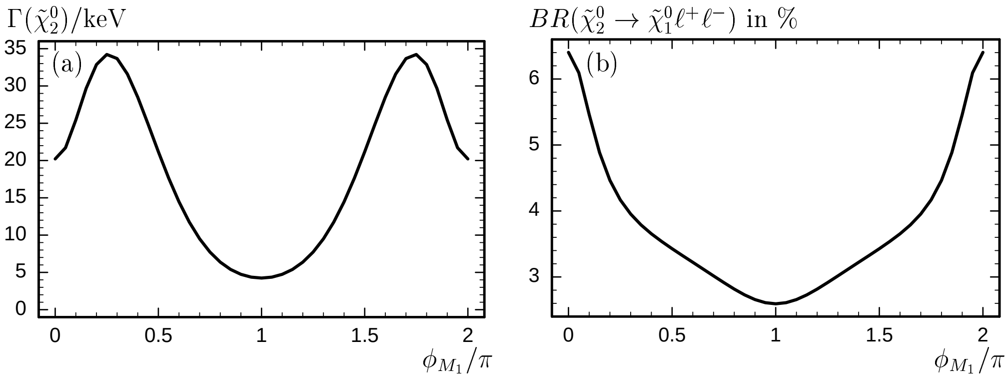

Figure 3: (a) Total decay width $\Gamma(\tilde{\chi}^0_2)$ and (b) branching ratio $BR(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \ell^+ \ell^-)$, $\ell = e$ or μ, in scenario A of table 1 for $\phi_\mu = 0$.

show the full phase dependence of the asymmetry and the cross section, we also give some examples where we vary ϕ_μ and ϕ_{M_1} in the whole range, relaxing the constraints from the EDMs.

4.1 $e^+e^- \to \tilde{\chi}^0_1 \tilde{\chi}^0_2$

First we discuss neutralino production $e^+e^- \to \tilde{\chi}^0_1 \tilde{\chi}^0_2$ with subsequent leptonic three-body decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 \ell^+ \ell^-$. In this case one obtains CP-violating contributions from both production and decay. In figure 4(a) we show the CP asymmetry A_T as a function of the phase ϕ_{M_1} for scenario A defined in table 1 for $\phi_\mu = 0$ and two centre of mass energies $\sqrt{s} = 350\,\text{GeV}$ and $\sqrt{s} = 500\,\text{GeV}$. The beam polarisations are fixed at $P_{e^-} = -0.8$ and $P_{e^+} = +0.6$. The CP asymmetry attains the largest values of about 11 % (−11 %) for $\sqrt{s} = 500\,\text{GeV}$ at $\phi_{M_1} \approx 0.2\pi$ ($\phi_{M_1} \approx 1.8\pi$). For $\sqrt{s} = 350\,\text{GeV}$, i.e. closer to threshold

JHEP08(2004)038

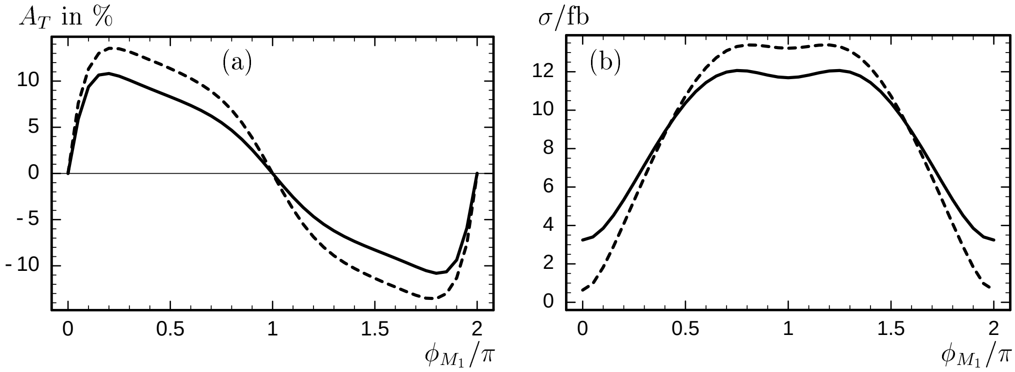

Figure 4: (a) CP asymmetry A_T (eq. (1.4)) and (b) cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0) \cdot BR(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\ell^+\ell^-)$, summed over $\ell = e, \mu$, in scenario A of table 1 for $\phi_\mu = 0$, $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ and $\sqrt{s} = 500\,\mathrm{GeV}$ (solid), $\sqrt{s} = 350\,\mathrm{GeV}$ (dashed).

of the production, A_T reaches values of $\pm 13.5\,\%$ at $\phi_{M_1} \approx 0.2\pi, 1.8\pi$. Choosing $\phi_\mu = 0.1\pi$ instead of $\phi_\mu = 0$ leaves the asymmetry A_T almost unchanged. Note that A_T does not have its largest values at $\phi_{M_1} = \frac{\pi}{2}$ ($\phi_{M_1} = \frac{3\pi}{2}$), where the imaginary parts of the respective couplings are maximal, but at smaller (larger) phases. The reason is an interplay between the ϕ_{M_1} dependence of the cross section, shown in figure 4(b), and the ϕ_{M_1} dependence of the numerator of A_T, eq. (3.2). This numerator is a sum of products of CP-odd and CP-even factors, which essentially have a $\sin\phi_{M_1}$ and $\cos\phi_{M_1}$ behaviour, respectively (for $\phi_\mu = 0$). We plot in figure 4(b) the cross section $\sigma = \sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0) \cdot BR(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\ell^+\ell^-)$ for the production and subsequent decay process $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\tilde{\chi}_1^0\ell^+\ell^-$ (eq. (2.28)), summed over $\ell = e, \mu$. Note that also the cross section has a rather strong ϕ_{M_1} dependence. At $\phi_{M_1} = \pi$ the production cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0)$ has a maximum, whereas the branching ratio $BR(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\ell^+\ell^-)$ has a minimum (see figure 3(b)) resulting in the dip of the cross section for the combined process of production and decay at $\phi_{M_1} = \pi$.

In figure 5 we show the contour lines of the CP asymmetry A_T in the ϕ_{M_1}-ϕ_μ plane in scenario A (table 1) for $\sqrt{s} = 500\,\mathrm{GeV}$ and two sets of beam polarisations $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ and $P_{e^-} = +0.8$, $P_{e^+} = -0.6$. In the scenario considered the ϕ_{M_1} dependence is stronger than that on ϕ_μ. In the case of $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ the largest asymmetries $|A_T| \approx 10\,\%$ are reached for $\phi_{M_1} \approx 0.2\pi, 1.8\pi$, rather independent of ϕ_μ (figure 5(a)). For $P_{e^-} = +0.8$, $P_{e^+} = -0.6$ (figure 5(b)) the maximal values of $|A_T| \approx 7.5\,\%$ are attained near $\phi_{M_1} \approx 0.3\pi, \phi_\mu \approx 0.4\pi$ and $\phi_{M_1} \approx 1.7\pi, \phi_\mu \approx 1.6\pi$. The sign flip of A_T between figure 5(a) and (b) and the weak ϕ_μ dependence follows from the structure of the $\tilde{\chi}_i^0\tilde{\ell}_{L,R}\ell$ couplings and the facts that $m_{\tilde{\ell}_R} < m_{\tilde{\ell}_L}$ and that the magnitude of the $\tilde{B}$ component of $\tilde{\chi}_2^0$ depends rather strongly on ϕ_{M_1}. For right-handed (left-handed) electrons the $\tilde{B}$ component ($\tilde{B}$ and $\tilde{W}^3$ components) of $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ contribute, which leads to a more pronounced ϕ_{M_1} dependence than that on ϕ_μ.

In figure 6(a)–(d) we show the contour lines of the CP asymmetry A_T and of the cross section for $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$, $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\ell^+\ell^-$, summed over $\ell = e, \mu$, in the M_2-$|\mu|$ plane for $|M_1|/M_2 = 5/3\tan^2\theta_W$, $\phi_{M_1} = 0.5\pi$ and $\phi_\mu = 0$, $\tan\beta = 10$, $m_{\tilde{\ell}_L} = 267.6\,\mathrm{GeV}$,

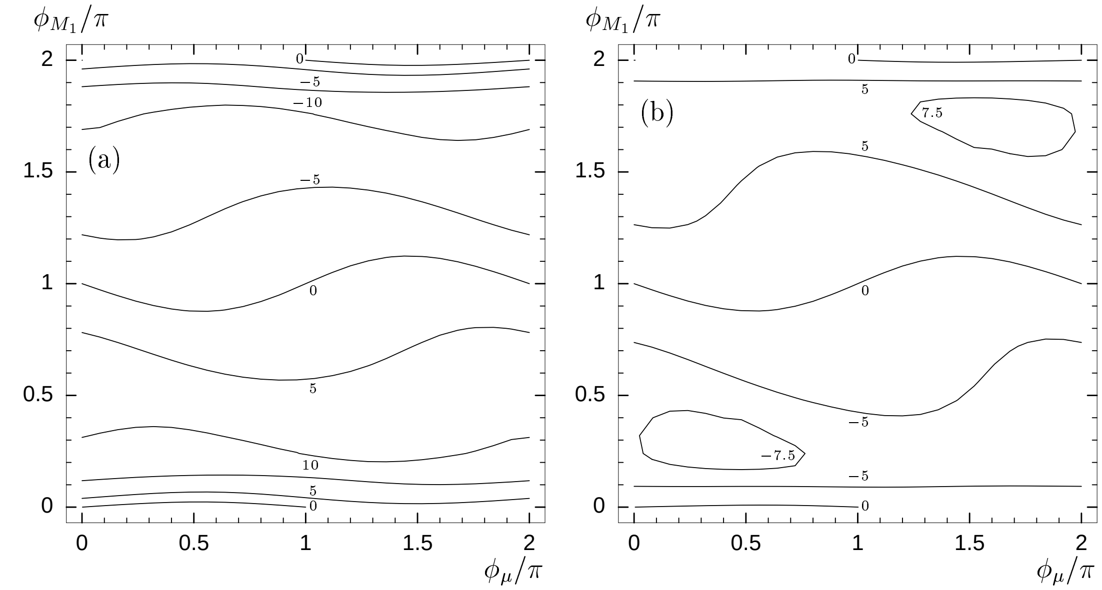

Figure 5: Contours of the CP asymmetry A_T (eq. (1.4)) in % for $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$ with subsequent decay $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\ell^+\ell^-$ in scenario A of table 1 for $\sqrt{s} = 500\,\text{GeV}$ and (a) $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ and (b) $P_{e^-} = +0.8$, $P_{e^+} = -0.6$.

$m_{\tilde{\ell}_R} = 224.4\,\text{GeV}$, at $\sqrt{s} = 500\,\text{GeV}$ and two sets of beam polarisations, $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ and $P_{e^-} = +0.8$, $P_{e^+} = -0.6$. For both polarisation configurations A_T is largest in the region $|\mu| \approx 240\,\text{GeV}$ and $M_2 \gtrsim 300\,\text{GeV}$ with maximal values $A_T \approx 7.5\,\%$ for $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ (figure 6(a)) and $A_T \approx -10\,\%$ for $P_{e^-} = +0.8$, $P_{e^+} = -0.6$ (figure 6(b)). For $P_{e^-} = -0.8$ ($P_{e^-} = +0.8$) the main contributions to the asymmetry A_T come from the Z-$\tilde{\ell}_L$ (Z-$\tilde{\ell}_R$) interference terms, therefore large asymmetries are attained if both neutralinos have significant higgsino and gaugino components. In figure 6(c) and (d) we show the cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\tilde{\chi}_1^0\ell^+\ell^-)$, summed over $\ell = e, \mu$. For $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ (figure 6(c)) the cross section is larger than about 10 fb in the region with maximal A_T ($|\mu| \approx 240\,\text{GeV}$, $M_2 \gtrsim 300\,\text{GeV}$). For $|\mu| \gtrsim 400\,\text{GeV}$ and $M_2 \approx 125\,\text{GeV}$ the cross section reaches values larger than 40 fb, the asymmetry, however, is rather small, $A_T \approx -2.5\,\%$. For $P_{e^-} = +0.8$, $P_{e^+} = -0.6$ (figure 6(d)) the cross section has values between 1 and 10 fb in the region where A_T is maximal ($|\mu| \approx 240\,\text{GeV}$, $M_2 \gtrsim 300\,\text{GeV}$).

In principle, also for the case of hadronic neutralino decays into heavy quarks, $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 b\bar{b}$ and $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 c\bar{c}$, it will be possible to measure the asymmetry A_T. We have calculated A_T in the M_2-$|\mu|$ plane and found values of roughly the same order of magnitude as those shown in figure 6 for the leptonic decay. The cross sections are larger by roughly a factor 5–10 in the main part of the parameter region of figure 6 and approximately of the same order of magnitude as in the leptonic case for $|\mu| \gtrsim 400\,\text{GeV}$. The experimental errors, however, are expected to be larger than for the leptonic $\tilde{\chi}_2^0$ decays, because the distinction of the b and $\bar{b}$ or c and $\bar{c}$ charges and the resolution of the jet momenta are worse than in the leptonic case [27].

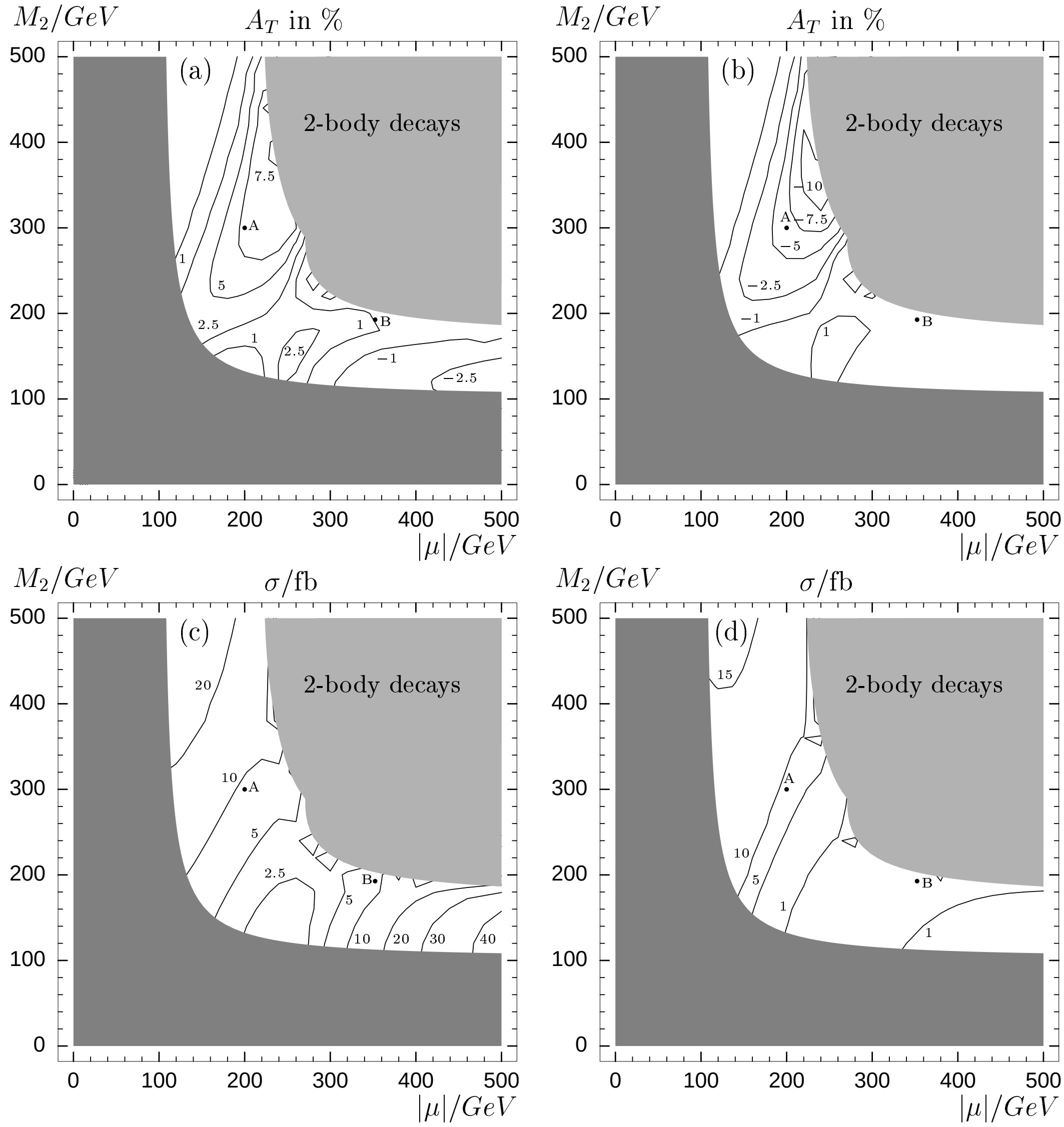

Figure 6: (a), (b) Contours of the CP asymmetry A_T (eq. (1.4)) in % for $e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2$ with subsequent decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1\ell^+\ell^-$ and (c), (d) contours of the corresponding cross section $\sigma(e^+e^- \to \tilde{\chi}^0_1\tilde{\chi}^0_2) \cdot BR(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1\ell^+\ell^-)$, summed over $\ell = e, \mu$, in fb, respectively, for $\tan\beta = 10$, $m_{\tilde{\ell}_L} = 267.6\,\mathrm{GeV}$, $m_{\tilde{\ell}_R} = 224.4\,\mathrm{GeV}$, $|M_1|/M_2 = 5/3\tan^2\theta_W$, $\phi_{M_1} = 0.5\pi$ and $\phi_\mu = 0$ with $\sqrt{s} = 500\,\mathrm{GeV}$ and (a), (c) $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ and (b), (d) $P_{e^-} = +0.8$, $P_{e^+} = -0.6$. The points mark scenarios A and B of Table 1. The dark shaded area marks the parameter space with $m_{\tilde{\chi}^\pm_1} < 103.5\,\mathrm{GeV}$ excluded by LEP. In the light shaded area the analysed three-body decay is strongly suppressed because $m_{\tilde{\chi}^0_2} > m_Z + m_{\tilde{\chi}^0_1}$ or $m_{\tilde{\chi}^0_2} > m_{\tilde{\ell}_R}$.

We also studied the $|M_1|$ dependence of the asymmetry A_T and the cross section, relaxing the GUT relation for $|M_1|$ with all other parameters as in scenario B of Table 1. In figure 7(a) we show the T-odd asymmetry A_T as a function of ϕ_{M_1} for four values of $|M_1| = 90\,\mathrm{GeV}$, $95\,\mathrm{GeV}$, $100\,\mathrm{GeV}$ and $105\,\mathrm{GeV}$. Here maximal asymmetries $|A_T| \approx 4\,\%$ are reached. For $|M_1| = 105\,\mathrm{GeV}$ A_T shows nearly two complete oscillations, whereas for

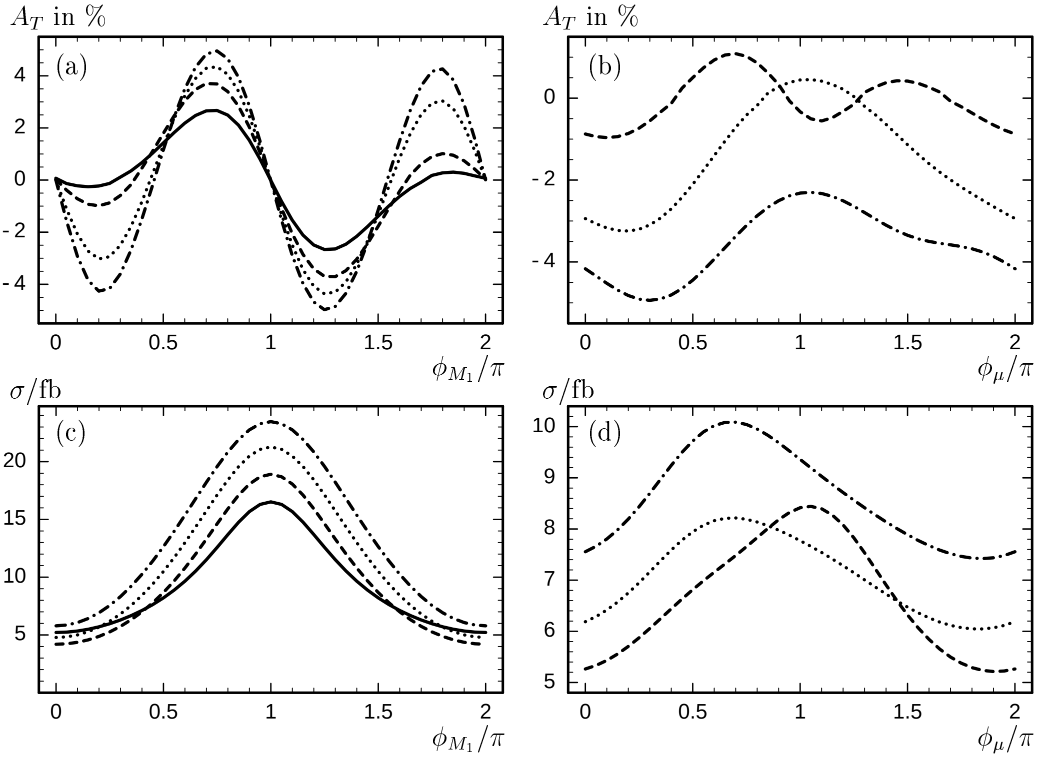

Figure 7: (a), (b) CP asymmetry A_T (eq. (1.4)) and (c), (d) cross section $\sigma(e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0) \cdot BR(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\ell^+\ell^-)$, summed over $\ell = e, \mu$, in scenario B of Table 1 with (a), (c) $\phi_\mu = 0$, (b), (d) $\phi_{M_1} = 0.25\pi$ and $|M_1| = 90\,\mathrm{GeV}$ (solid), $|M_1| = 95\,\mathrm{GeV}$ (dashed), $|M_1| = 100\,\mathrm{GeV}$ (dotted) and $|M_1| = 105\,\mathrm{GeV}$ (dashdotted) for $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ and $\sqrt{s} = 500\,\mathrm{GeV}$.

$|M_1| = 90\,\mathrm{GeV}$ there is only one oscillation with extrema at $\phi_{M_1} = 0.75\pi$ and 1.25π. In both cases this behaviour is caused by cancellations between the contributions $\Sigma_P^{a,\mathrm{O}}(\tilde{\chi}_i^0)\Sigma_D^{a,\mathrm{E}}(\tilde{\chi}_i^0)$ from production and $\Sigma_P^{a,\mathrm{E}}(\tilde{\chi}_i^0)\Sigma_D^{a,\mathrm{O}}(\tilde{\chi}_i^0)$ from decay in eq. (3.2), which have opposite sign. For $|M_1| = 90\,\mathrm{GeV}$ the contributions from the decay are dominant, whereas for $|M_1| = 105\,\mathrm{GeV}$ both contributions are of similar magnitude, resulting in two oscillations. The corresponding cross sections are shown in figure 7(c). In figure 7(b) we show A_T as a function of ϕ_μ for $\phi_{M_1} = 0.25\pi$ and three values of $|M_1| = 95\,\mathrm{GeV}$, $100\,\mathrm{GeV}$ and $105\,\mathrm{GeV}$. (We do not discuss $|M_1| = 90\,\mathrm{GeV}$ because in this case $m_{\tilde{\chi}_2^0} > m_{\tilde{\chi}_1^0} + m_{Z^0}$ for $0.5\pi \lesssim \phi_\mu \lesssim 1.7\pi$.) For $|M_1| = 100\,\mathrm{GeV}$ A_T exhibits the strongest variation with ϕ_μ with maximum $|A_T| \approx 3\,\%$ for $\phi_\mu \approx 0.2\pi$ and minimum $|A_T| \approx 0$ for $\phi_\mu \approx \pi$. The corresponding cross sections are shown in figure 7(d).

Furthermore we have analysed the dependence of the asymmetry A_T on the masses of the right and left sleptons, assuming $m_{\tilde{\ell}_R}$ and $m_{\tilde{\ell}_L}$ are free parameters, for $\phi_{M_1} = 0.5\pi$ and $\phi_\mu = 0$ with the other parameters as in scenario A of table 1. For $\sqrt{s} = 500\,\mathrm{GeV}$ and beam polarisations $P_{e^-} = -0.8$ and $P_{e^+} = +0.6$ we find that the asymmetry A_T is larger than $6\,\%$ for $m_{\tilde{\ell}_R} \approx 200\,\mathrm{GeV}$ and $m_{\tilde{\ell}_L} \gtrsim 150\,\mathrm{GeV}$. For $m_{\tilde{\ell}_L} \approx 200\,\mathrm{GeV}$ and $m_{\tilde{\ell}_R} \gtrsim 300\,\mathrm{GeV}$ the asymmetry is smaller than $-6\,\%$. This is due to the fact, that for these beam polarisations the contributions from $\tilde{\ell}_L$ exchange to the CP asymmetry are dominant. Outside of these narrow regions around $m_{\tilde{\ell}_R} \approx 200\,\mathrm{GeV}$ or $m_{\tilde{\ell}_L} \approx 200\,\mathrm{GeV}$ the asymmetry is very small. For opposite beam polarisations $P_{e^-} = +0.8$, $P_{e^+} = -0.6$ the $\tilde{\ell}_R$ contributions to A_T are dominant and the sign of the asymmetry changes: for $m_{\tilde{\ell}_R} \approx 200\,\mathrm{GeV}$ and $m_{\tilde{\ell}_L} \gtrsim 250\,\mathrm{GeV}$ it is $A_T \lesssim -11\,\%$ and for $m_{\tilde{\ell}_L} \approx 200\,\mathrm{GeV}$ and $300 \lesssim m_{\tilde{\ell}_R} \lesssim 850\,\mathrm{GeV}$ it is $A_T \gtrsim 6\,\%$.

4.2 $e^+e^- \to \tilde{\chi}_2^0\tilde{\chi}_2^0$

In the reaction $e^+e^- \to \tilde{\chi}_2^0\tilde{\chi}_2^0$ there are only T-odd contributions from the decay, i.e. from the second term in eq. (3.2). There is no T-odd contribution from the production, because the amplitudes eqs. (2.10)–(2.14) are real and the first term in eq. (3.2) vanishes. We will give numerical results for the case where one neutralino decays leptonically and the other decays hadronically. If both neutralinos would decay into lepton pairs of the same flavour, it may be experimentally difficult to distinguish the two lepton pairs coming from different neutralinos. In figure 8 we show the contour lines in the M_2-$|\mu|$ plane of the CP asymmetry A_T and of the cross section with one neutralino decaying leptonically, $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0\ell^+\ell^-$, summed over $\ell = e, \mu$, and the other neutralino decaying hadronically, $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 q\bar{q}$, summed over $q = u, d, s, c, b$, for $\sqrt{s} = 350\,\mathrm{GeV}$ and $P_{e^-} = -0.8$, $P_{e^+} = +0.6$. The other parameters are $\tan\beta = 10$, $m_{\tilde{\ell}_L} = 267.6\,\mathrm{GeV}$, $m_{\tilde{\ell}_R} = 224.4\,\mathrm{GeV}$, $|M_1|/M_2 = 5/3\tan^2\theta_W$, $\phi_{M_1} = 0.5\pi$ and $\phi_\mu = 0$. For this process A_T is considerably smaller than for $e^+e^- \to \tilde{\chi}_1^0\tilde{\chi}_2^0$, because for production of a pair of equal neutralinos only the decay amplitudes contribute to the asymmetry. The largest asymmetries are obtained for $|\mu| \gtrsim 400\,\mathrm{GeV}$ and $M_2 \approx 150\,\mathrm{GeV}$, where $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ have gaugino character. The maximum values are $A_T \approx 1.8\,\%$. The cross section, however, reaches values up to 10 fb in this region. The hadronic branching ratio $\sum_{q=u,d,s,c,b} BR(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 q\bar{q})$ is between $50\,\%$ and $95\,\%$ in the main part of the parameter region of figure 8 and between $30\,\%$ and $50\,\%$ for $|\mu| \gtrsim 400\,\mathrm{GeV}$. For $\sqrt{s} = 500\,\mathrm{GeV}$ the asymmetries are smaller with maximum of $0.7\,\%$ for $|\mu| \gtrsim 400\,\mathrm{GeV}$ and $M_2 \approx 150\,\mathrm{GeV}$.

We want to remark that it may also be possible to distinguish the two lepton pairs, if one neutralino decays to an e^+e^- pair, the other one to a $\mu^+\mu^-$ pair. Similarly it may be possible to distinguish the two quark pairs, if one neutralino decays to a $b\bar{b}$ pair and the other one to a $c\bar{c}$ pair [27] or pair of light quarks. However, in the case of both neutralinos decaying into lepton pairs the event rate is expected to be very low, whereas in the case of the decay into quark pairs of different flavours the experimental errors are expected to be much larger.

4.3 $e^+e^- \to \tilde{\chi}_3^0\tilde{\chi}_2^0$

In figure 9 we show the contour lines of the CP asymmetry A_T in the ϕ_{M_1}-ϕ_μ plane in scenario A (Table 1) for $\sqrt{s} = 500\,\mathrm{GeV}$ and two sets of beam polarisations $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ and $P_{e^-} = +0.8$, $P_{e^+} = -0.6$. As in the case of $\tilde{\chi}_1^0\tilde{\chi}_2^0$ production (figure 5) the ϕ_{M_1} dependence is stronger than that on ϕ_μ in this scenario, since the $\tilde{B}$ component of the decaying particle $\tilde{\chi}_2^0$ depends stronger on ϕ_{M_1}. The largest asymmetries of $|A_T| \gtrsim 9\,\%$ are reached in the case of $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ (figure 9(a)) for $\phi_{M_1} \approx 0.25\pi, 1.75\pi$ and $0.6\pi \lesssim \phi_\mu \lesssim 1.4\pi$. In the case of $P_{e^-} = +0.8$, $P_{e^+} = -0.6$ (figure 9(b)) the sign of A_T flips and the largest asymmetries $|A_T| \gtrsim 7\,\%$ appear for $\phi_{M_1} \approx 0.25\pi, 1.75\pi$ and $0.8\pi \lesssim \phi_\mu \lesssim 1.2\pi$.

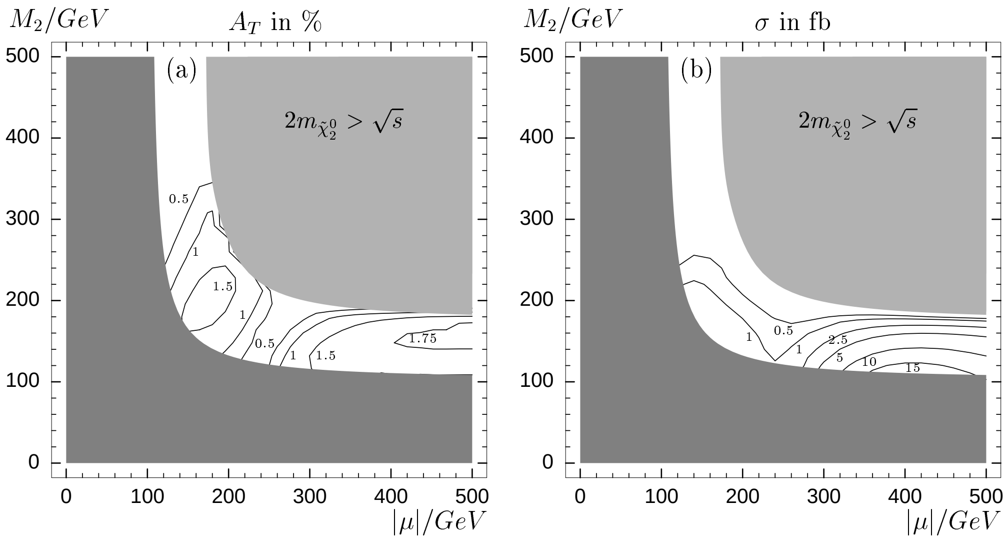

Figure 8: Contours (a) of the CP asymmetry A_T (eq. (1.4)) in % for $e^+e^- \to \tilde{\chi}^0_2\tilde{\chi}^0_2$ with subsequent decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1\ell^+\ell^-$ and (b) of the corresponding cross section $\sigma(e^+e^- \to \tilde{\chi}^0_2\tilde{\chi}^0_2) \cdot BR(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1\ell^+\ell^-) \cdot BR(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1 q\bar{q})$, summed over $\ell = e, \mu$ and $q = u, d, s, c, b$, in fb, respectively, for $\tan\beta = 10$, $m_{\tilde{\ell}_L} = 267.6\,\mathrm{GeV}$, $m_{\tilde{\ell}_R} = 224.4\,\mathrm{GeV}$, $|M_1|/M_2 = 5/3\tan^2\theta_W$, $\phi_{M_1} = 0.5\pi$ and $\phi_\mu = 0$ with $\sqrt{s} = 350\,\mathrm{GeV}$ and $P_{e^-} = -0.8$, $P_{e^+} = +0.6$. The dark shaded area marks the parameter space with $m_{\tilde{\chi}^\pm_1} < 103.5\,\mathrm{GeV}$ excluded by LEP. The light shaded area is kinematically not accessible.

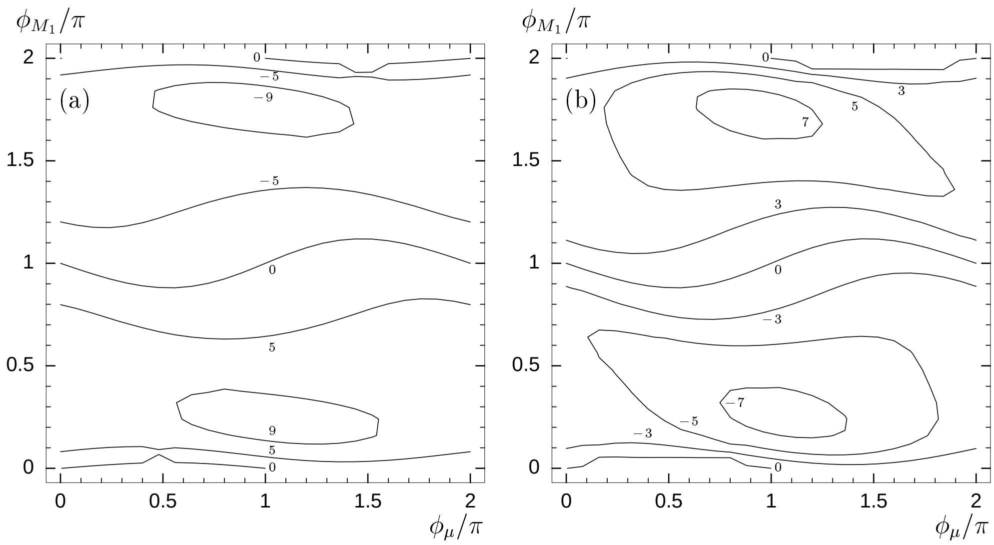

Figure 9: Contours of the CP asymmetry A_T (eq. (1.4)) in % for $e^+e^- \to \tilde{\chi}^0_3\tilde{\chi}^0_2$ with subsequent decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1\ell^+\ell^-$ in scenario A of table 1 for $\sqrt{s} = 500\,\mathrm{GeV}$ and(a) $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ and (b) $P_{e^-} = +0.8$, $P_{e^+} = -0.6$.

In figure 10 we show the contour lines of the CP asymmetry A_T and of the cross section for the process $e^+e^- \to \tilde{\chi}^0_3\tilde{\chi}^0_2$ with the subsequent decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1\ell^+\ell^-$, summed over $\ell = e, \mu$, in the M_2-$|\mu|$ plane for $\tan\beta = 10$, $m_{\tilde{\ell}_L} = 267.6\,\mathrm{GeV}$, $m_{\tilde{\ell}_R} = 224.4\,\mathrm{GeV}$,

$|M_1|/M_2 = 5/3\tan^2\theta_W$, $\phi_{M_1} = 0.5\pi$ and $\phi_\mu = 0$ for $\sqrt{s} = 500\,\mathrm{GeV}$ and two sets of beam polarisations $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ and $P_{e^-} = +0.8$, $P_{e^+} = -0.6$. $|A_T|$ has two maxima in the regions $|\mu| \approx 240\,\mathrm{GeV}$, $M_2 \approx 300\,\mathrm{GeV}$ and $|\mu| \approx 350\,\mathrm{GeV}$, $M_2 \approx 140\,\mathrm{GeV}$, respectively, for both polarisation configurations. The maximum values are $A_T \gtrsim 10\,\%$ for $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ (figure 10(a)) and $A_T \lesssim -8\,\%$ for $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ (figure 10(b)). The cross section reaches values larger than 10 fb (6 fb) in the region with maximal asymmetry around $|\mu| \approx 240\,\mathrm{GeV}$, $M_2 \approx 300\,\mathrm{GeV}$ for $P_{e^-} = -0.8$, $P_{e^+} = +0.6$, figure 10(c) ($P_{e^-} = +0.8$, $P_{e^+} = -0.6$, figure 10(d)). In the region around $|\mu| \approx 350\,\mathrm{GeV}$, $M_2 \approx 140\,\mathrm{GeV}$ the cross section is $\lesssim 1$ fb.

We have also calculated the branching ratio $BR(\tilde{\chi}^0_3 \to \tilde{\chi}^0_1 \ell^+\ell^-)$, $\ell = e$ or μ, of $\tilde{\chi}^0_3$. In parameter regions, where $\tilde{\chi}^0_3$ decays via a three-body decay and where it is especially difficult to distinguish the two lepton pairs coming from different neutralinos, $BR(\tilde{\chi}^0_3 \to \tilde{\chi}^0_1 \ell^+\ell^-)$ is always smaller than about 7 %.

Furthermore we have calculated the CP asymmetry A_T for the the process $e^+e^- \to \tilde{\chi}^0_4\tilde{\chi}^0_2$ with the subsequent decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1\ell^+\ell^-$, $\ell = e, \mu$. For the associated production of $\tilde{\chi}^0_2$ and $\tilde{\chi}^0_4$ the accessible parameter space at a linear collider with $\sqrt{s} = 500\,\mathrm{GeV}$ is rather small and is approximately constrained by $M_2 + |\mu| \lesssim 500\,\mathrm{GeV}$. For the same parameters as in figure 10 we find that the CP asymmetry has maximum values $|A_T| \approx 6\,\%$. However, in parameter regions with $|A_T| > 2\,\%$ the cross section $\sigma(e^+e^- \to \tilde{\chi}^0_4\tilde{\chi}^0_2) \cdot BR(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1\ell^+\ell^-)$, summed over $\ell = e, \mu$, is always smaller than 1 fb.

5. Conclusions

In this paper we have studied CP violation in neutralino production and subsequent three-body decay processes. As CP-sensitive observable we have chosen a T-odd asymmetry based on triple product correlations between the incoming and outgoing particles. A non-vanishing value for the asymmetry indicates directly CP-violating effects. We have provided compact analytical formulae for the cross section of the whole process as well as for the T-odd asymmetry for longitudinal beam polarisations. It is necessary to include the full spin correlations between production and decay. The asymmetry can be directly measured in the experiment without reconstruction of the momentum of the decaying neutralino or further final-state analyses. In our numerical results we have chosen representative scenarios and have shown that this T-odd asymmetry can reach values up to 13 % in some parts of the parameter space of the unconstrained MSSM, even for small CP-violating phases as may be indicated by the EDM constraints. This observable will therefore be an important tool in the search for CP-violating effects in the neutralino sector and the determination of the phases of the complex SUSY parameters.

Acknowledgments

We thank W. Porod for providing us a version of SPheno for complex SUSY parameters, prior to its release, which has been used to compute the neutralino decay widths. We thank T. Kernreiter, O. Kittel and W. Majerotto for useful discussions. This work is

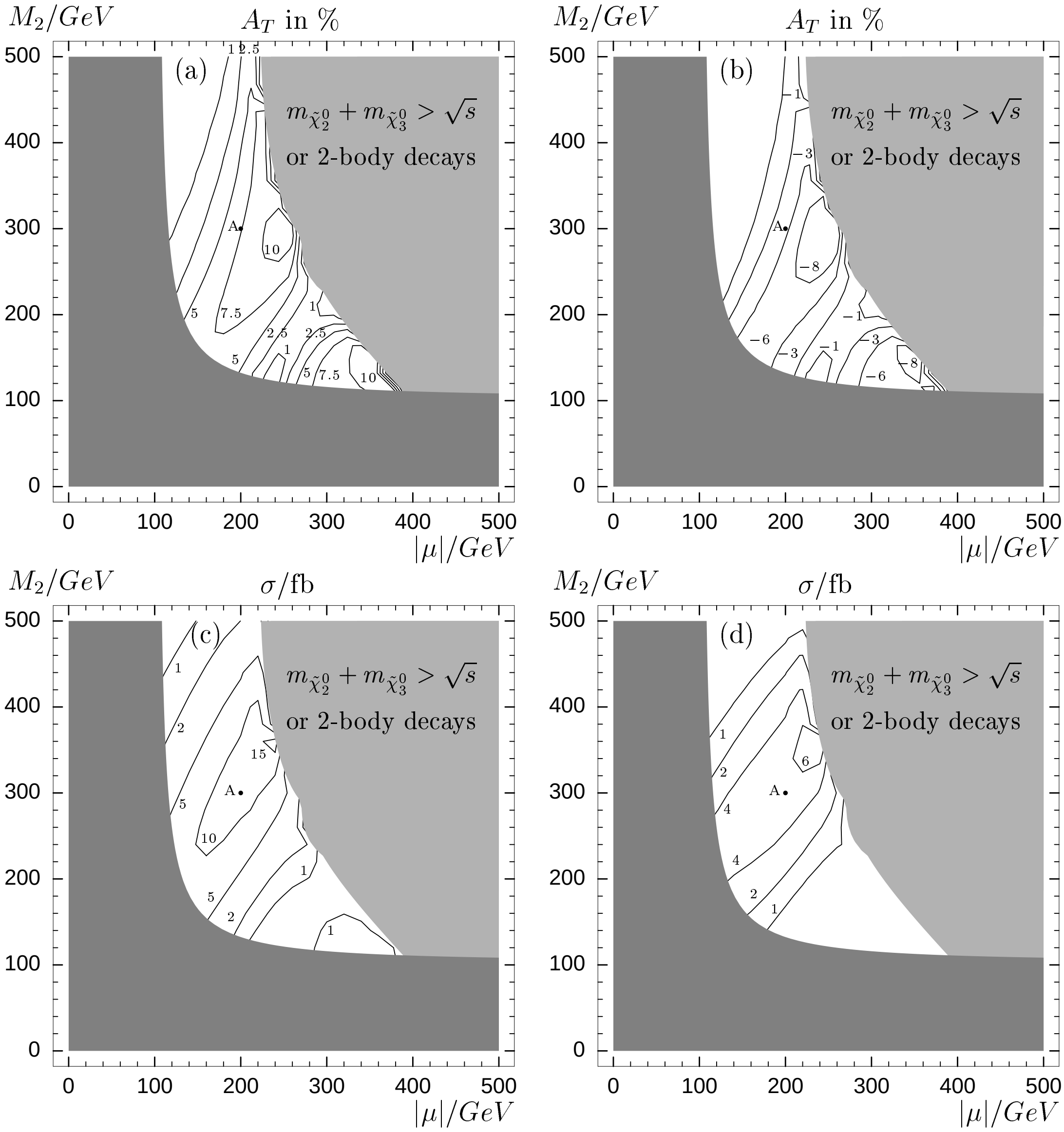

Figure 10: (a), (b) Contours of the CP asymmetry A_T (eq. (1.4)) in % for $e^+e^- \to \tilde{\chi}^0_3\tilde{\chi}^0_2$ with subsequent decay $\tilde{\chi}^0_2 \to \tilde{\chi}^0_1\ell^+\ell^-$ and (c), (d) contours of the corresponding cross section $\sigma(e^+e^- \to \tilde{\chi}^0_3\tilde{\chi}^0_2) \cdot BR(\tilde{\chi}^0_2 \to \tilde{\chi}^0_1\ell^+\ell^-)$, summed over $\ell = e, \mu$, in fb, respectively, for $\tan\beta = 10$, $m_{\tilde{\ell}_L} = 267.6\,\mathrm{GeV}$, $m_{\tilde{\ell}_R} = 224.4\,\mathrm{GeV}$, $|M_1|/M_2 = 5/3\tan^2\theta_W$, $\phi_{M_1} = 0.5\pi$ and $\phi_\mu = 0$ with $\sqrt{s} = 500\,\mathrm{GeV}$ and (a), (c) $P_{e^-} = -0.8$, $P_{e^+} = +0.6$ and (b), (d) $P_{e^-} = +0.8$, $P_{e^+} = -0.6$. The dark shaded area marks the parameter space with $m_{\tilde{\chi}^\pm_1} < 103.5\,\mathrm{GeV}$ excluded by LEP. The light shaded area is kinematically not accessible or in this area the analysed three-body decay is strongly suppressed because $m_{\tilde{\chi}^0_2} > m_Z + m_{\tilde{\chi}^0_1}$ or $m_{\tilde{\chi}^0_2} > m_{\tilde{\ell}_R}$, respectively.

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A. Kinematics

We choose a coordinate frame in the laboratory system, where the momenta are

$$p_1 = E_b(1, -\sin\Theta, 0, \cos\Theta)\,, \tag{A.1}$$

$$p_2 = E_b(1, \sin\Theta, 0, -\cos\Theta)\,, \tag{A.2}$$

$$p_3 = (E_i, 0, 0, -q)\,, \tag{A.3}$$

$$p_4 = (E_j, 0, 0, q)\,. \tag{A.4}$$

Θ is the scattering angle between the incoming $e^-(p_1)$ beam and the outgoing neutralino $\tilde{\chi}^0_j(p_4)$, the azimuth Φ can be chosen equal to zero. The energy and the momenta of the neutralinos $\tilde{\chi}^0_j(p_4)$ and $\tilde{\chi}^0_i(p_3)$ are

$$E_i = \frac{s + m_i^2 - m_j^2}{2\sqrt{s}}\,, \qquad E_j = \frac{s + m_j^2 - m_i^2}{2\sqrt{s}}\,, \qquad q = \frac{\sqrt{\lambda(s, m_i^2, m_j^2)}}{2\sqrt{s}}\,, \tag{A.5}$$

where m_i, m_j are the masses of the neutralinos and λ is the kinematical triangle function $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$.

The polarisation vectors $s^a_\mu(\tilde{\chi}^0_i)$ and $s^b_\mu(\tilde{\chi}^0_j)$ $(a, b = 1, 2, 3)$ of the neutralinos in the laboratory system are

$$s^{1\mu}(\tilde{\chi}^0_{i,j}) = (0, \mp 1, 0, 0)\,, \tag{A.6}$$

$$s^{2\mu}(\tilde{\chi}^0_{i,j}) = (0, 0, 1, 0)\,, \tag{A.7}$$

$$s^{3\mu}(\tilde{\chi}^0_{i,j}) = \frac{1}{m_{i,j}}(q, 0, 0, \mp E_{i,j})\,, \tag{A.8}$$

where $s^3(\tilde{\chi}^0_{i,j})$ describes the longitudinal polarisation, $s^1(\tilde{\chi}^0_{i,j})$ the transverse polarisation in the scattering plane, and $s^2(\tilde{\chi}^0_{i,j})$ the transverse polarisation perpendicular to the scattering plane.

B. Explicit expressions of the amplitude squared

B.1 Production

Here we give the analytic expressions for the production spin density matrix. The terms

$$P(\tilde{\chi}^0_i \tilde{\chi}^0_j) = P(ZZ) + P(Z\tilde{e}_L) + P(Z\tilde{e}_R) + P(\tilde{e}_L\tilde{e}_L) + P(\tilde{e}_R\tilde{e}_R) \tag{B.1}$$

are independent of the polarisation of the neutralinos [16, 28],

$$\begin{aligned} P(ZZ) = {} & \frac{g^4}{\cos^4\Theta_W}|\Delta^s(Z)|^2(R_\ell^2 c_R + L_\ell^2 c_L) \times \\ & \times \left\{ |O''^L_{ij}|^2(f_1 + f_2) - [(\mathrm{Re}O''^L_{ij})^2 - (\mathrm{Im}O''^L_{ij})^2] f_3 \right\}, \end{aligned} \tag{B.2}$$

$$P(Z\tilde{e}_L) = \frac{g^4}{2\cos^2\Theta_W} L_\ell c_L \times$$
$$\times \mathrm{Re}\Big\{\Delta^s(Z)\Big[-(\Delta^{t*}(\tilde{e}_L) f^{L*}_{\ell i} f^L_{\ell j} O''^{L*}_{ij} + \Delta^{u*}(\tilde{e}_L) f^L_{\ell i} f^{L*}_{\ell j} O''^{L}_{ij}) f_3 +$$
$$+ 2\Delta^{t*}(\tilde{e}_L) f^{L*}_{\ell i} f^L_{\ell j} O''^{L}_{ij} f_1 + 2\Delta^{u*}(\tilde{e}_L) f^L_{\ell i} f^{L*}_{\ell j} O''^{L*}_{ij} f_2\Big]\Big\}, \tag{B.3}$$
$$P(\tilde{e}_L\tilde{e}_L) = \frac{g^4}{4} c_L \Big\{|f^L_{\ell i}|^2 |f^L_{\ell j}|^2 (|\Delta^t(\tilde{e}_L)|^2 f_1 + |\Delta^u(\tilde{e}_L)|^2 f_2) -$$
$$- \mathrm{Re}\{(f^{L*}_{\ell i})^2 (f^L_{\ell j})^2 \Delta^u(\tilde{e}_L)\Delta^{t*}(\tilde{e}_L) f_3\}\Big\}. \tag{B.4}$$

$P(Z\tilde{e}_R)$, $P(\tilde{e}_R\tilde{e}_R)$ are obtained by the following substitutions in eqs. (B.3), (B.4):

$$L_\ell \to R_\ell, \quad c_L \to c_R, \quad \Delta^{t,u}(\tilde{e}_L) \to \Delta^{t,u}(\tilde{e}_R), \quad O''^{L}_{ij} \to O''^{R}_{ij}, \quad f^L_{\ell i,j} \to f^R_{\ell i,j}. \tag{B.5}$$

The beam polarisation weighting factors are given by

$$c_L = (1 - P_{e^-})(1 + P_{e^+}), \tag{B.6}$$
$$c_R = (1 + P_{e^-})(1 - P_{e^+}) \tag{B.7}$$

and the kinematical factors in invariant form are

$$f_1 = (p_1 p_4)(p_2 p_3), \tag{B.8}$$
$$f_2 = (p_1 p_3)(p_2 p_4), \tag{B.9}$$
$$f_3 = m_i m_j (p_1 p_2). \tag{B.10}$$

The terms

$$\Sigma^a_P(\tilde{\chi}^0_i) = \Sigma^a_P(ZZ) + \Sigma^a_P(Z\tilde{e}_L) + \Sigma^a_P(Z\tilde{e}_R) + \Sigma^a_P(\tilde{e}_L\tilde{e}_L) + \Sigma^a_P(\tilde{e}_R\tilde{e}_R), \tag{B.11}$$

which depend on the polarisation of the neutralino $\tilde{\chi}^0_i$ and which are relevant for the calculation of the CP asymmetries, are given by

$$\Sigma^a_P(ZZ) = \frac{g^4}{\cos^4\Theta_W} |\Delta^s(Z)|^2 (R^2_\ell c_R - L^2_\ell c_L) \times \tag{B.12}$$
$$\times \Big\{|O''^{L}_{ij}|^2 (f^a_2 - f^a_1) - [(\mathrm{Re}O''^{L}_{ij})^2 - (\mathrm{Im}O''^{L}_{ij})^2] f^a_3 + 2(\mathrm{Re}O''^{L}_{ij})(\mathrm{Im}O''^{L}_{ij}) i f^a_4\Big\},$$
$$\Sigma^a_P(Z\tilde{e}_L) = \frac{g^4}{2\cos^2\Theta_W} L_\ell c_L \times$$
$$\times \mathrm{Re}\Big\{\Delta^s(Z)\Big[2 f^L_{\ell i} f^{L*}_{\ell j} O''^{L*}_{ij} \Delta^{u*}(\tilde{e}_L) f^a_1 - 2 f^{L*}_{\ell i} f^L_{\ell j} O''^{L}_{ij} \Delta^{t*}(\tilde{e}_L) f^a_2 +$$
$$+ \Big[f^L_{\ell i} f^{L*}_{\ell j} O''^{L}_{ij} \Delta^{u*}(\tilde{e}_L) + f^{L*}_{\ell i} f^L_{\ell j} O''^{L*}_{ij} \Delta^{t*}(\tilde{e}_L)\Big] f^a_3 +$$
$$+ \Big[f^{L*}_{\ell i} f^L_{\ell j} O''^{L*}_{ij} \Delta^{t*}(\tilde{e}_L) - f^L_{\ell i} f^{L*}_{\ell j} O''^{L}_{ij} \Delta^{u*}(\tilde{e}_L)\Big] f^a_4\Big]\Big\}, \tag{B.13}$$
$$\Sigma^a_P(\tilde{e}_L\tilde{e}_L) = \frac{g^4}{4} c_L \Big[|f^L_{\ell i}|^2 |f^L_{\ell j}|^2 (|\Delta^u(\tilde{e}_L)|^2 f^a_1 - |\Delta^t(\tilde{e}_L)|^2 f^a_2) +$$
$$+ \mathrm{Re}\{(f^{L*}_{\ell i})^2 (f^L_{\ell j})^2 \Delta^u(\tilde{e}_L)\Delta^{t*}(\tilde{e}_L)(f^a_3 + f^a_4)\}\Big]. \tag{B.14}$$

$\Sigma^a_P(Z\tilde{e}_R), \Sigma^a_P(\tilde{e}_R\tilde{e}_R)$ are obtained by the substitutions eq.(B.5) and

$$f^a_{1,2,3} \to -f^a_{1,2,3} \qquad \text{and} \qquad f^a_4 \to f^a_4 \tag{B.15}$$

in eqs. (B.13), (B.14), where the kinematical factors in invariant form are given by

$$f^a_1 = m_i (p_2 p_4)(p_1 s^a)\,, \tag{B.16}$$
$$f^a_2 = m_i (p_1 p_4)(p_2 s^a)\,, \tag{B.17}$$
$$f^a_3 = m_j \left[(p_1 p_3)(p_2 s^a) - (p_2 p_3)(p_1 s^a)\right], \tag{B.18}$$
$$f^a_4 = i m_j \epsilon_{\mu\nu\rho\sigma} p^\mu_2 p^\nu_1 s^{a\rho} p^\sigma_3\,. \tag{B.19}$$

The polarisation vectors $s^a(\tilde{\chi}^0_i)$ of the neutralino $\tilde{\chi}^0_i$ are given in the laboratory system by eqs. (A.6)–(A.8).

B.2 Leptonic 3-body decay

Here we give the analytical expressions for the different contributions to the decay density matrix for the three-body decay $\tilde{\chi}^0_i(p_3) \to \tilde{\chi}^0_k(p_5)\ell^+(p_6)\ell^-(p_7)$, where we sum over the spins of the final-state particles [16, 28]. The contributions independent of the polarisation of the neutralino $\tilde{\chi}^0_i$ are

$$D(\tilde{\chi}^0_i) = D(ZZ) + D(Z\tilde{\ell}_L) + D(Z\tilde{\ell}_R) + D(\tilde{\ell}_L\tilde{\ell}_L) + D(\tilde{\ell}_R\tilde{\ell}_R) \tag{B.20}$$

with

$$\begin{aligned} D(ZZ) &= 8\frac{g^4}{\cos^4\Theta_W}|\Delta^{s_i}(Z)|^2(L^2_\ell + R^2_\ell) \times \\ &\quad \times \left[|O''^L_{ki}|^2(g_1+g_2) + [(\mathrm{Re}O''^L_{ki})^2 - (\mathrm{Im}O''^L_{ki})^2]g_3\right], \end{aligned} \tag{B.21}$$

$$\begin{aligned} D(Z\tilde{\ell}_L) &= 4\frac{g^4}{\cos^2\Theta_W}L_\ell \mathrm{Re}\Big\{\Delta^{s_i}(Z)\Big[f^L_{\ell i}f^{L*}_{\ell k}\Delta^{t_i *}(\tilde{\ell}_L)(2O''^L_{ki}g_1 + O''^{L*}_{ki}g_3) + \\ &\qquad + f^{L*}_{\ell i}f^L_{\ell k}\Delta^{u_i *}(\tilde{\ell}_L)(2O''^{L*}_{ki}g_2 + O''^L_{ki}g_3)\Big]\Big\}, \end{aligned} \tag{B.22}$$

$$\begin{aligned} D(\tilde{\ell}_L\tilde{\ell}_L) &= 2g^4\Big[|f^L_{\ell i}|^2|f^L_{\ell k}|^2(|\Delta^{t_i}(\tilde{\ell}_L)|^2 g_1 + |\Delta^{u_i}(\tilde{\ell}_L)|^2 g_2) \\ &\qquad + \mathrm{Re}\left\{(f^{L*}_{\ell i})^2(f^L_{\ell k})^2\Delta^{t_i}(\tilde{\ell}_L)\Delta^{u_i *}(\tilde{\ell}_L)\right\}g_3\Big]. \end{aligned} \tag{B.23}$$

The quantities $D(Z\tilde{\ell}_R), D(\tilde{\ell}_R\tilde{\ell}_R)$ can be derived from eqs. (B.22), (B.23) by the substitutions

$$L_\ell \to R_\ell\,, \qquad \Delta^{t_i,u_i}(\tilde{\ell}_L) \to \Delta^{t_i,u_i}(\tilde{\ell}_R)\,, \qquad O''^L_{ki} \to O''^R_{ki}\,, \qquad f^L_{\ell i,k} \to f^R_{\ell i,k}\,. \tag{B.24}$$

The kinematical factors are

$$g_1 = (p_5 p_7)(p_3 p_6)\,, \tag{B.25}$$
$$g_2 = (p_5 p_6)(p_3 p_7)\,, \tag{B.26}$$
$$g_3 = m_i m_k (p_6 p_7)\,. \tag{B.27}$$

JHEP08(2004)038

The contributions which depend on the polarisation of the decaying neutralino $\tilde{\chi}_i^0$ are

$$\Sigma_D^a(\tilde{\chi}_i^0) = \Sigma_D^a(ZZ) + \Sigma_D^a(Z\tilde{\ell}_L) + \Sigma_D^a(Z\tilde{\ell}_R) + \Sigma_D^a(\tilde{\ell}_L\tilde{\ell}_L) + \Sigma_D^a(\tilde{\ell}_R\tilde{\ell}_R) \tag{B.28}$$

with[1]

$$\begin{aligned}
\Sigma_D^a(ZZ) &= 8\frac{g^4}{\cos^4\Theta_W}|\Delta^{s_i}(Z)|^2(R_\ell^2 - L_\ell^2) \times \\
&\quad\times\Big[-[(\mathrm{Re}O''^L_{ki})^2 - (\mathrm{Im}O''^L_{ki})^2]g_3^a + |O''^L_{ki}|^2(g_1^a - g_2^a) - \\
&\qquad - 2Re(O''^L_{ki})\mathrm{Im}(O''^L_{ki})ig_4^a\Big], \qquad\qquad (B.29)\\
\Sigma_D^a(Z\tilde{\ell}_L) &= \frac{4g^4}{\cos^2\Theta_W}L_\ell\mathrm{Re}\Big\{\Delta^{s_i}(Z)\Big[f_{\ell i}^L f_{\ell k}^{L*}\Delta^{t_i *}(\tilde{\ell}_L)\Big(-2O''^L_{ki}g_1^a + O''^{L*}_{ki}(g_3^a - g_4^a)\Big) + \qquad (B.30)\\
&\qquad + f_{\ell i}^{L*} f_{\ell k}^L\Delta^{u_i *}(\tilde{\ell}_L)(2O''^{L*}_{ki}g_2^a + O''^L_{ki}(g_3^a + g_4^a))\Big]\Big\},\\
\Sigma_D^a(\tilde{\ell}_L\tilde{\ell}_L) &= 2g^4\Big[|f_{\ell i}^L|^2|f_{\ell k}^L|^2[|\Delta^{u_i}(\tilde{\ell}_L)|^2 g_2^a - |\Delta^{t_i}(\tilde{\ell}_L)|^2 g_1^a] + \\
&\qquad + \mathrm{Re}\Big\{(f_{\ell i}^{L*})^2(f_{\ell k}^L)^2\Delta^{t_i}(\tilde{\ell}_L)\Delta^{u_i *}(\tilde{\ell}_L)(g_3^a + g_4^a)\Big\}\Big]. \qquad\qquad (B.31)
\end{aligned}$$

The contributions $\Sigma_D^a(Z\tilde{\ell}_R), \Sigma_D^a(\tilde{\ell}_R\tilde{\ell}_R)$ are obtained from eqs. (B.30), (B.31) by applying the substitutions in eq. (B.24) and in addition

$$g_{1,2,3}^a \to -g_{1,2,3}^a\,, \qquad g_4^a \to g_4^a\,. \tag{B.32}$$

The kinematical factors are

$$\begin{aligned}
g_1^a &= m_i(p_5p_7)(p_6s^a)\,, &(B.33)\\
g_2^a &= m_i(p_5p_6)(p_7s^a)\,, &(B.34)\\
g_3^a &= m_k[(p_3p_6)(p_7s^a) - (p_3p_7)(p_6s^a)]\,, &(B.35)\\
g_4^a &= im_k\epsilon_{\mu\nu\rho\sigma}s^{a\mu}p_3^\nu p_7^\rho p_6^\sigma\,. &(B.36)
\end{aligned}$$

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JHEP08(2004)038

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JHEP08(2004)038